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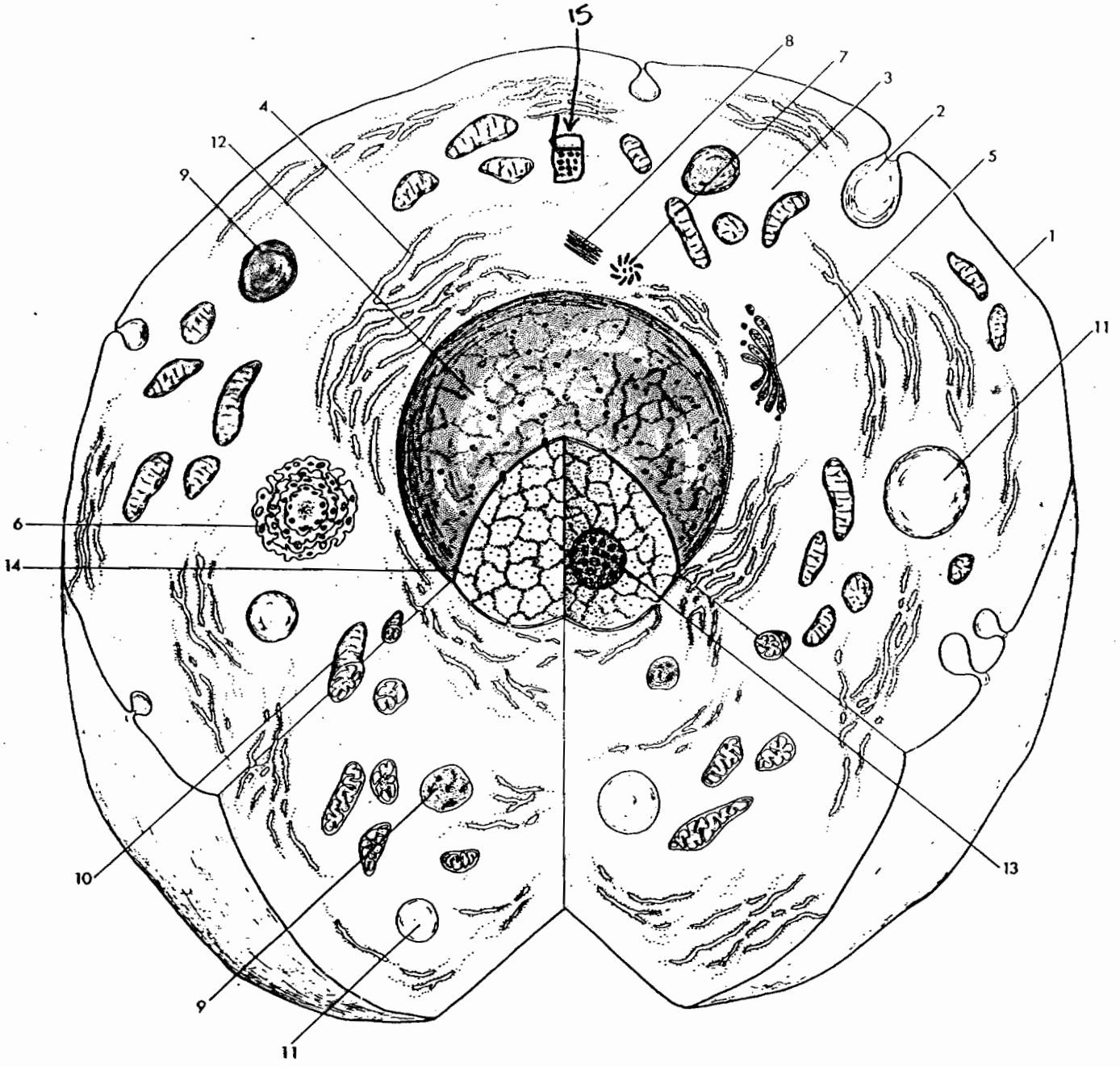
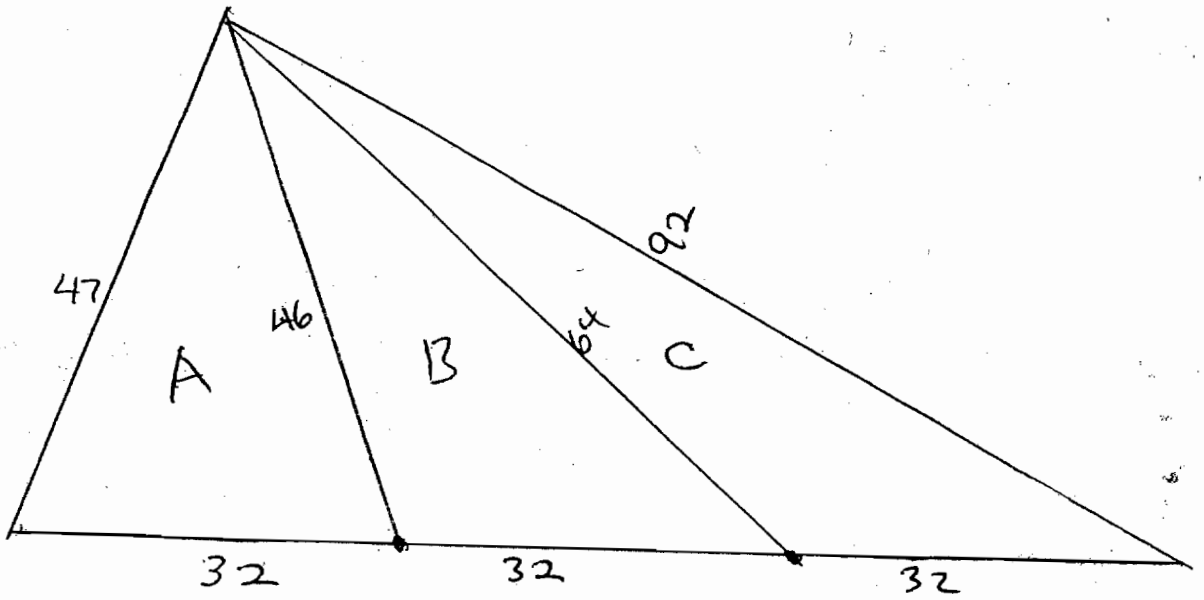
FROM: Dick Smith

# ANIMAL CELL Evolution

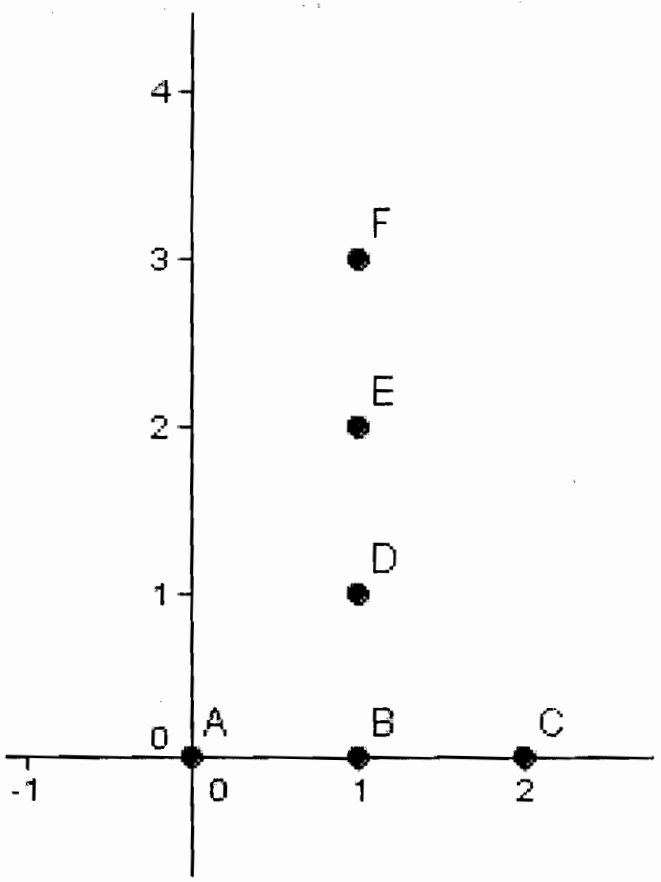



1. Cell membrane
2. Pinocytic vesicle
3. Cytoplasm
4. Endoplasmic reticulum
5. Golgi body (section)
6. Golgi body
7. Centrosome (c.s.)
8. Centrosome (l.s.)
9. Lysosome
10. Mitochondrion
11. Vacuole
12. Nucleus
13. Nucleolus
14. Nuclear membrane
15. cell phone

Carolina Biological Supply Company, Burlington, North Carolina 27215




Which Triangle has the Largest Area ?




ABC BCD CDE DEF

ABD BCE CDF

ABE BCF CEF

ABF BDE

ACD BDF

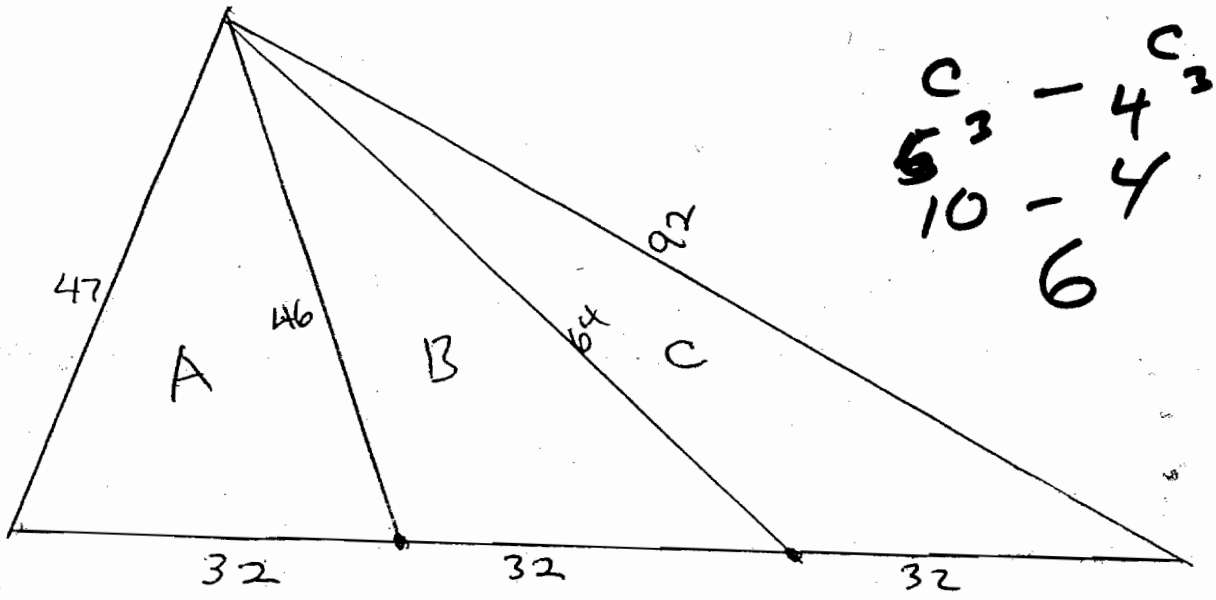
ACE BEF

ACF

ADE

ADF

AEF



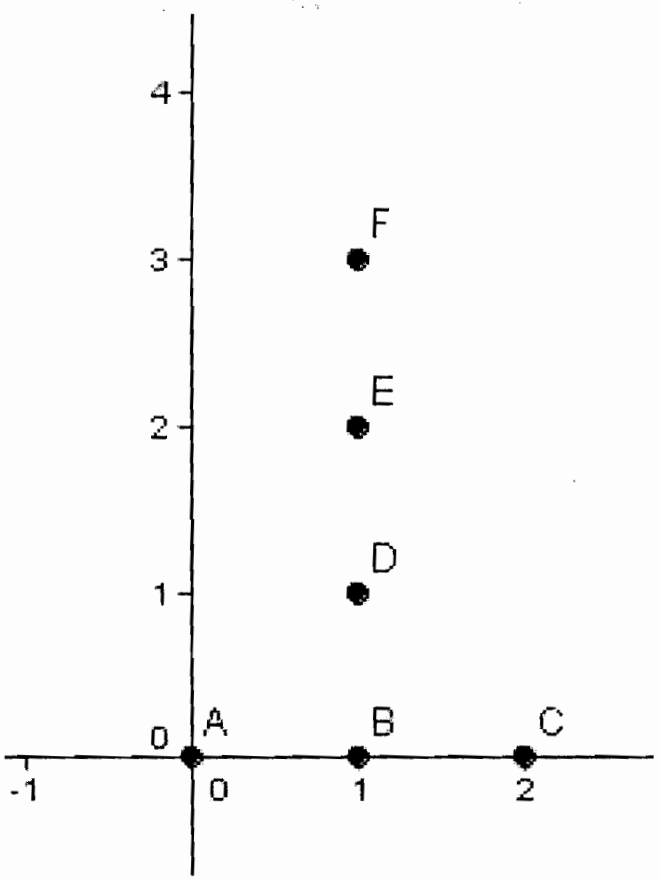

$C_3 - 4^{C_3}$

$5^3 - 4$

$10 - 4$

$6$

Which Triangle has the Largest Area ?




ABC BCD CDE DEF

ABD BCE CDF

ABE BCF CEF

ABF BDE

ACD BDF

ACE BEF

ACF

ADE

ADF

AEF

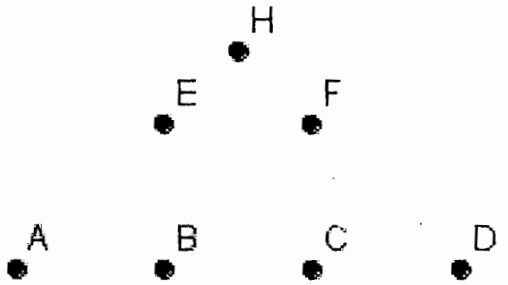
$C^{C_3} - C_3$

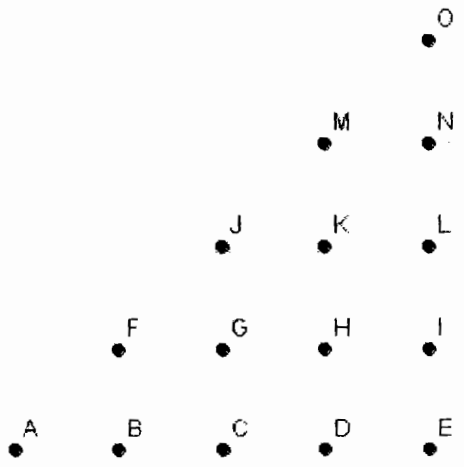
$6^{C_3} - 4^3 - 3$

$20 - 4 - 1$

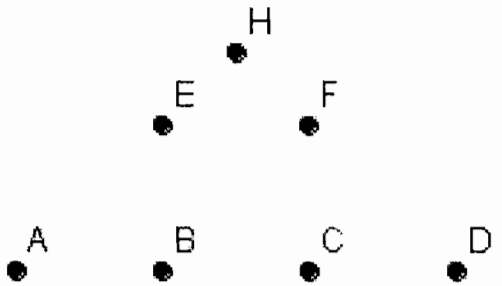
$15$

How many triangles?



How many triangles?

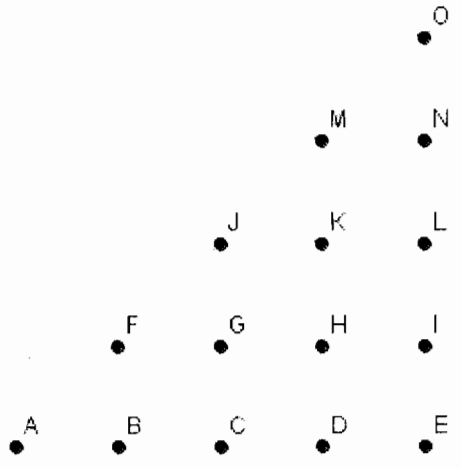



$$\binom{8}{3} - 4\binom{3}{3} - 2\binom{3}{3}$$

$$56 - 4 - 2$$

$$29$$

---





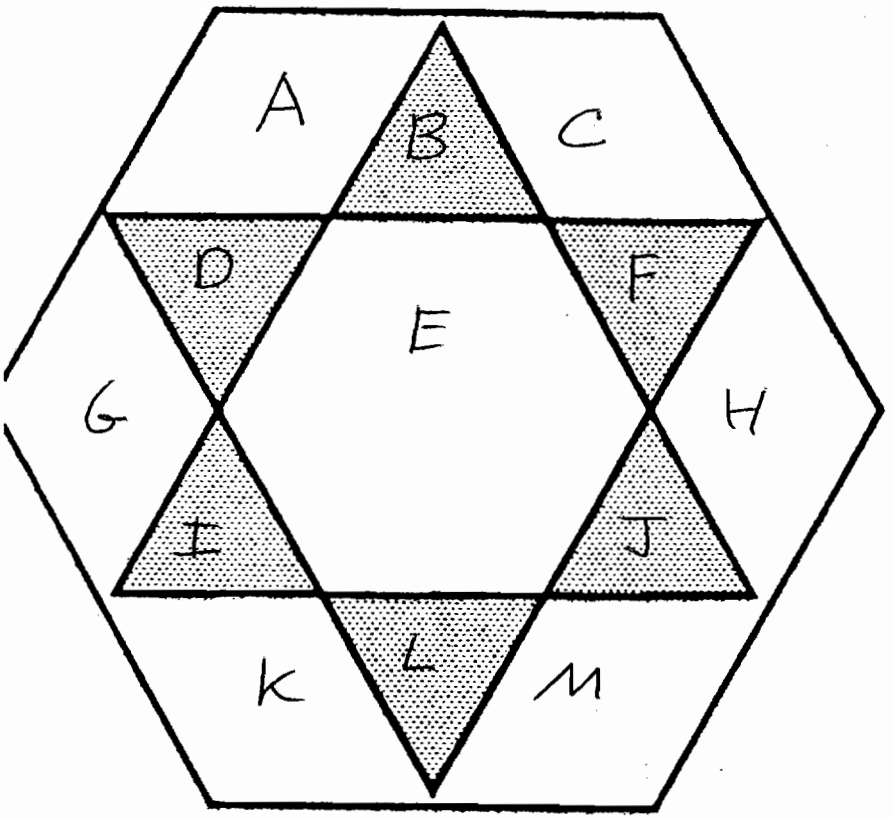

$$\binom{15}{3} - 2\binom{5}{3} - 2\binom{4}{3} - 2\binom{3}{3}$$

$$455 - 20 - 8 - 2$$

$$425$$

(A)

Rind the Probability of "landing in each of the Regions.




$A = C = G = H = K = M$

$B = D = F = I = J = L$

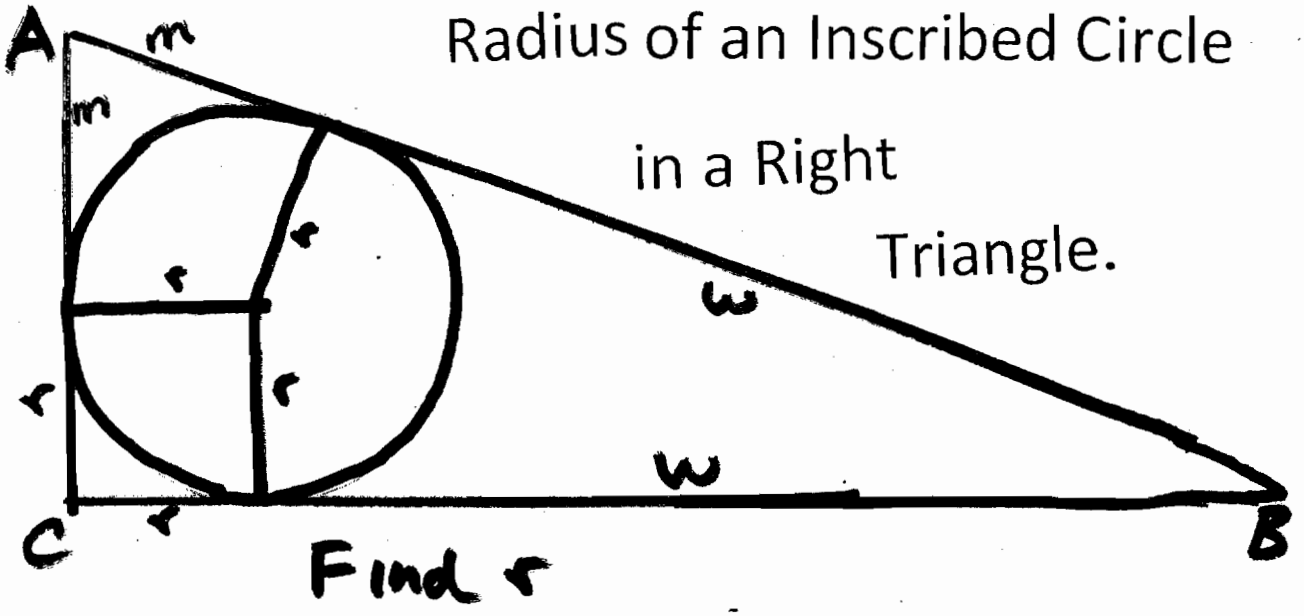
$A = 2B$ and $E = 6B$

shapes like B $= \frac{1}{24} \times 6 = \frac{1}{4}$

shapes like A $= \frac{1}{12} \times 6 = \frac{1}{2}$

$E = \frac{1}{4}$

# Radius of an Inscribed Circle in a Right Triangle.




AB=c  Ac=b  BC=a

w+m=c , r+w=a , r+m=b so w=a-r and m=b-r

b-r+a-r=c

-2r = c-a-b and r= $\frac{a+b-c}{2}$

Another formula

$c^2 = a^2 + b^2$

$c^2$ + 2ab = $a^2 + 2ab + b^2$

$c^2$ + 2ab = $a^2 + 2ab + b^2$

2ab = $(a + b)^2$ - $c^2$

2ab= (a+b+c)(a+b-c)

$\frac{ab}{a+b+c}=\frac{a+b-c}{2}$ = r by Transitivity

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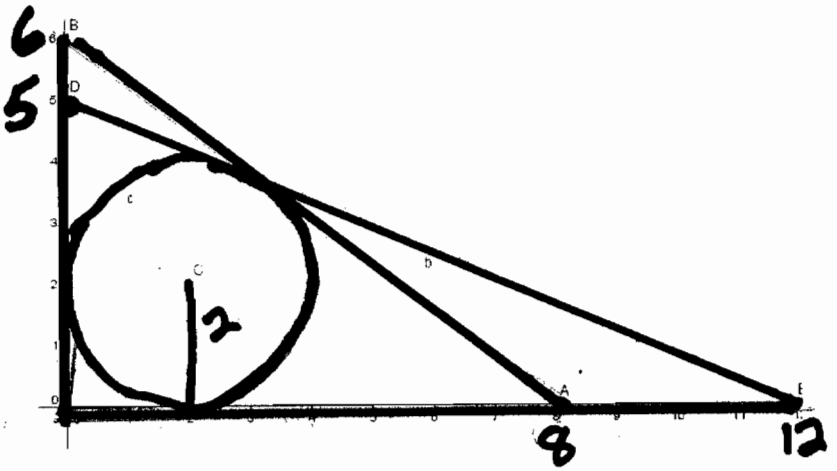
A) If a=6, b=8 and c=10

$r=\frac{(6)(8)}{6+8+10}=\frac{6+8-10}{2}=2$

B) If a=5, b=12 and c=13

$r=\frac{(6)(12)}{5+12+13}=\frac{5+12-13}{2}=2$




Compute leg b when leg a and radius r are known

$r=\frac{a+b-c}{2}$

$2r=a+b-\sqrt{a^2+b^2}$

$2r-a-b=-\sqrt{a^2+b^2}$

$4r^2-2ar-2rb-2ar+a^2+ab-2rb+ab+b^2=a^2+b^2$

$-4ar-4rb+2ab+4r^2=0$

$2b(-2r+a)=4ar-4r^2$

$b=\frac{2ar-2r^2}{a-2r}$

| a | b | r | A |
|---|---|---|---|
| 6 | 8 | 2 | 24 |
| 5.9 | 8.211 | 2 | 24.2211 |
| 5.8 | 8.444 | 2 | 24.4889 |
| 5.7 | 8.706 | 2 | 24.8118 |
| 5.6 | 9 | 2 | 25.2 |
| 5.5 | 9.333 | 2 | 25.6667 |
| 5.4 | 9.714 | 2 | 26.2286 |
| 5.3 | 10.15 | 2 | 26.9077 |
| 5.2 | 10.67 | 2 | 27.7333 |
| 5.1 | 11.27 | 2 | 28.7455 |
| 5 | 12 | 2 | 30 |
| 4.9 | 12.89 | 2 | 31.5778 |
| 4.8 | 14 | 2 | 33.6 |
| 4.7 | 15.43 | 2 | 36.2571 |
| 4.6 | 17.33 | 2 | 39.8667 |
| 4.5 | 20 | 2 | 45 |
| 4.4 | 24 | 2 | 52.8 |
| 4.3 | 30.67 | 2 | 65.9333 |
| 4.2 | 44 | 2 | 92.4 |
| 4.1 | 84 | 2 | 172.2 |
| 4.09 | 92.89 | 2 | 189.958 |
| 4.08 | 104 | 2 | 212.16 |
| 4.07 | 118.3 | 2 | 240.711 |
| 4.06 | 137.3 | 2 | 278.787 |
| 4.05 | 164 | 2 | 332.1 |
| 4.04 | 204 | 2 | 412.08 |
| 4.03 | 270.7 | 2 | 545.393 |
| 4.02 | 404 | 2 | 812.04 |
| 4.01 | 804 | 2 | 1612.02 |
| 4 | infinity | 2 | no triangle |

## Ratios

A Circle with Radius 2 inscribed in a 6-8-10 Right Triangle

$$\frac{\text{Circle}}{\text{Triangle}} = \frac{4\pi}{24} = \frac{\pi}{6} \approx .1\overline{6}\,\pi$$

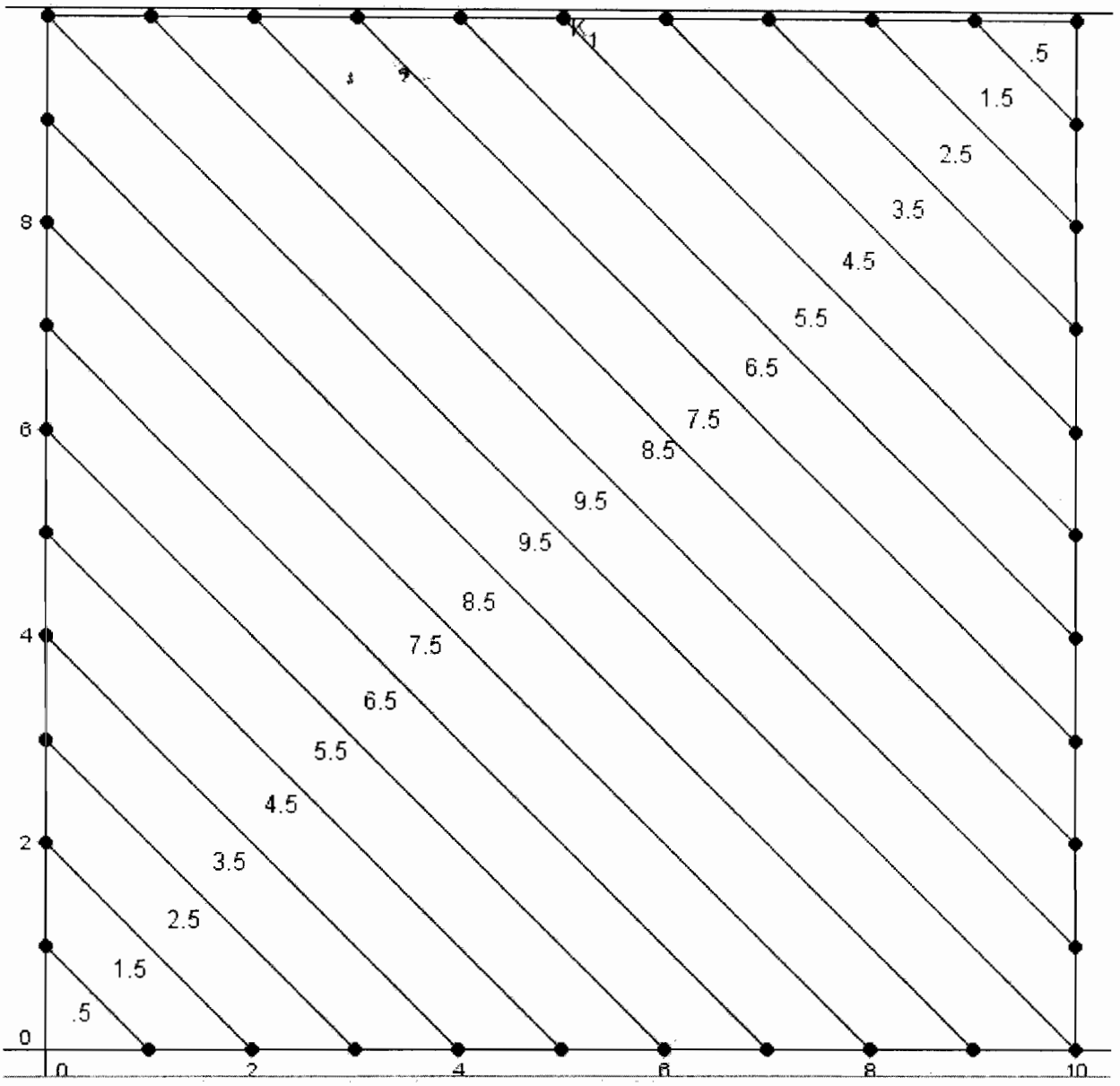
A Circle with Radius 2 inscribed in a 5-12-13 Right Triangle

$$\frac{\text{Circle}}{\text{Triangle}} = \frac{4\pi}{30} = \frac{2\pi}{15} \approx .1\overline{3}\,\pi$$

A Circle with Radius 2 inscribed in a 6.828-6.828-9.656 Isosceles Right Triangle

$$\frac{\text{Circle}}{\text{Triangle}} = \frac{4\pi}{23.314} \approx .17156\,\pi \approx .539$$

(max value of ratio)




$$Sum = \frac{20}{2}(.5 + 9.5)$$

$$10(10)$$

$$100$$

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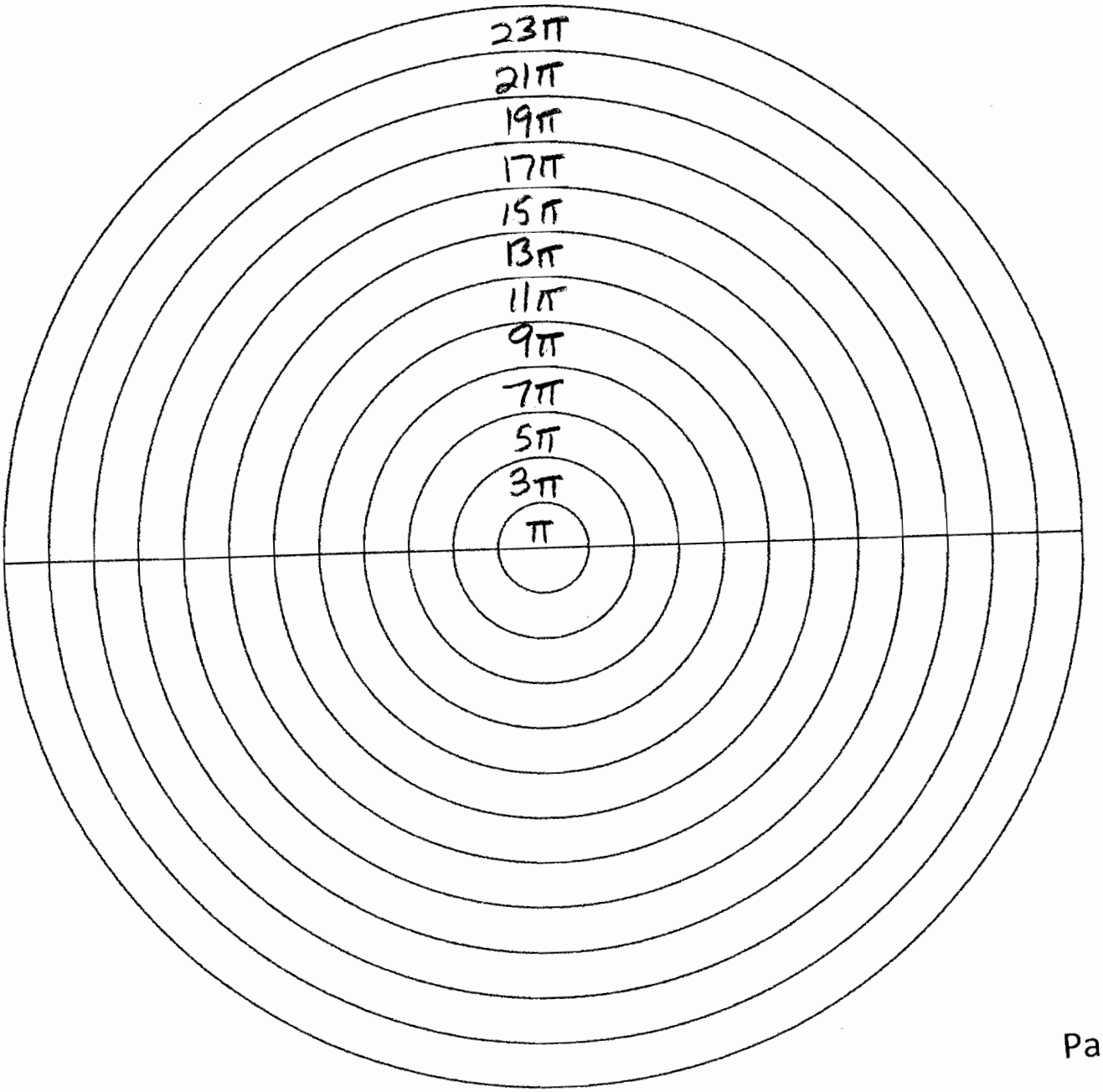





$A = \pi \cdot 12^2 = 144\pi$

or

$A = \frac{12}{2}(\pi + 23\pi) = 144\pi$

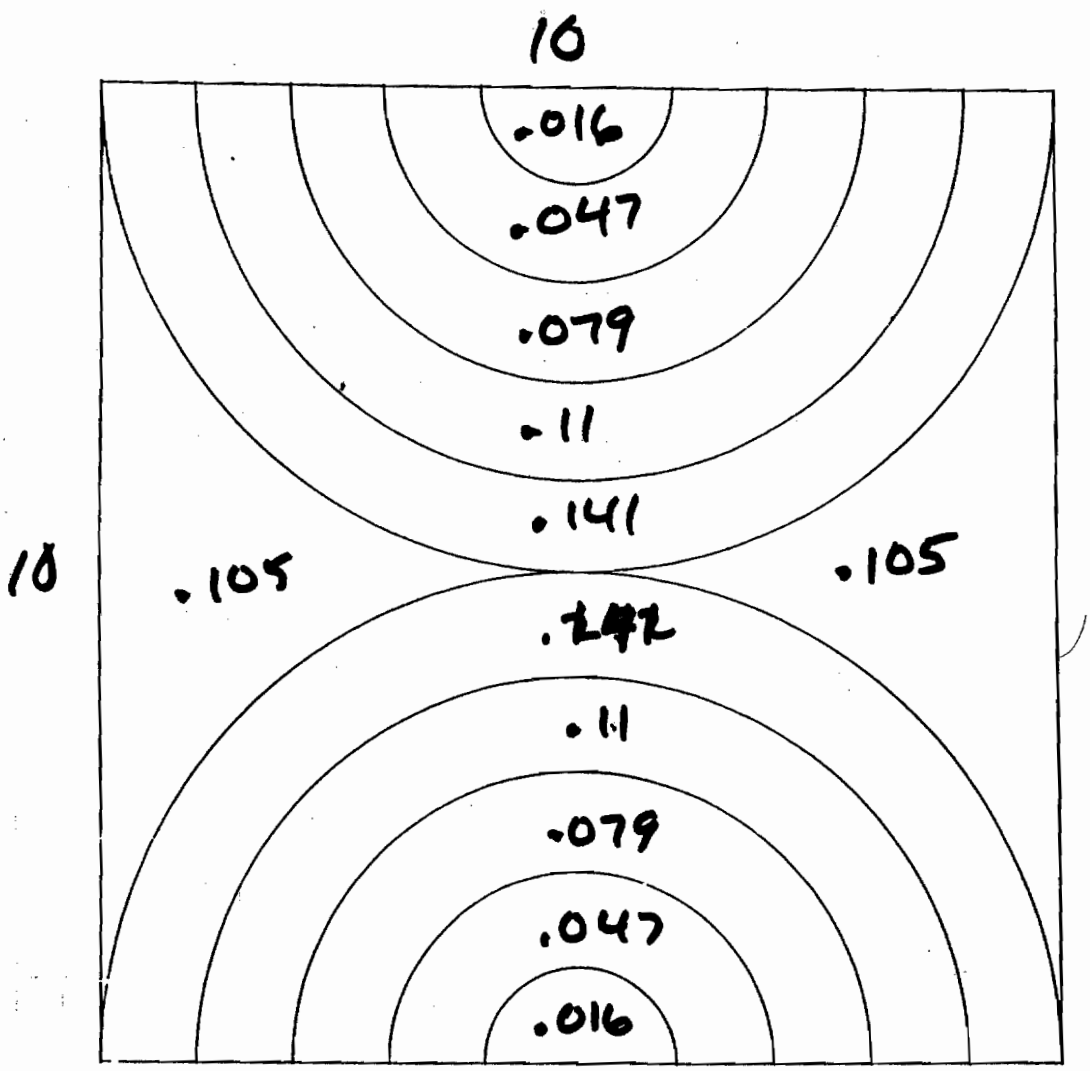
Circle rings

| RADIUS | AREA | SHELL | PROB |
|---|---|---|---|
| 1 | 3.14159 | 3.14159 | 0.006944 |
| 2 | 12.56636 | 9.42477 | 0.020833 |
| 3 | 28.27431 | 15.70795 | 0.034722 |
| 4 | 50.26544 | 21.99113 | 0.048611 |
| 5 | 78.53975 | 28.27431 | 0.0625 |
| 6 | 113.0972 | 34.55749 | 0.076389 |
| 7 | 153.9379 | 40.84067 | 0.090278 |
| 8 | 201.0618 | 47.12385 | 0.104167 |
| 9 | 254.4688 | 53.40703 | 0.118056 |
| 10 | 314.159 | 59.69021 | 0.131944 |
| 11 | 380.1324 | 65.97339 | 0.145833 |
| 12 | 452.389 | 72.25657 | 0.159722 |
| | | | 1 |

$\sqrt{72} = 8.49$

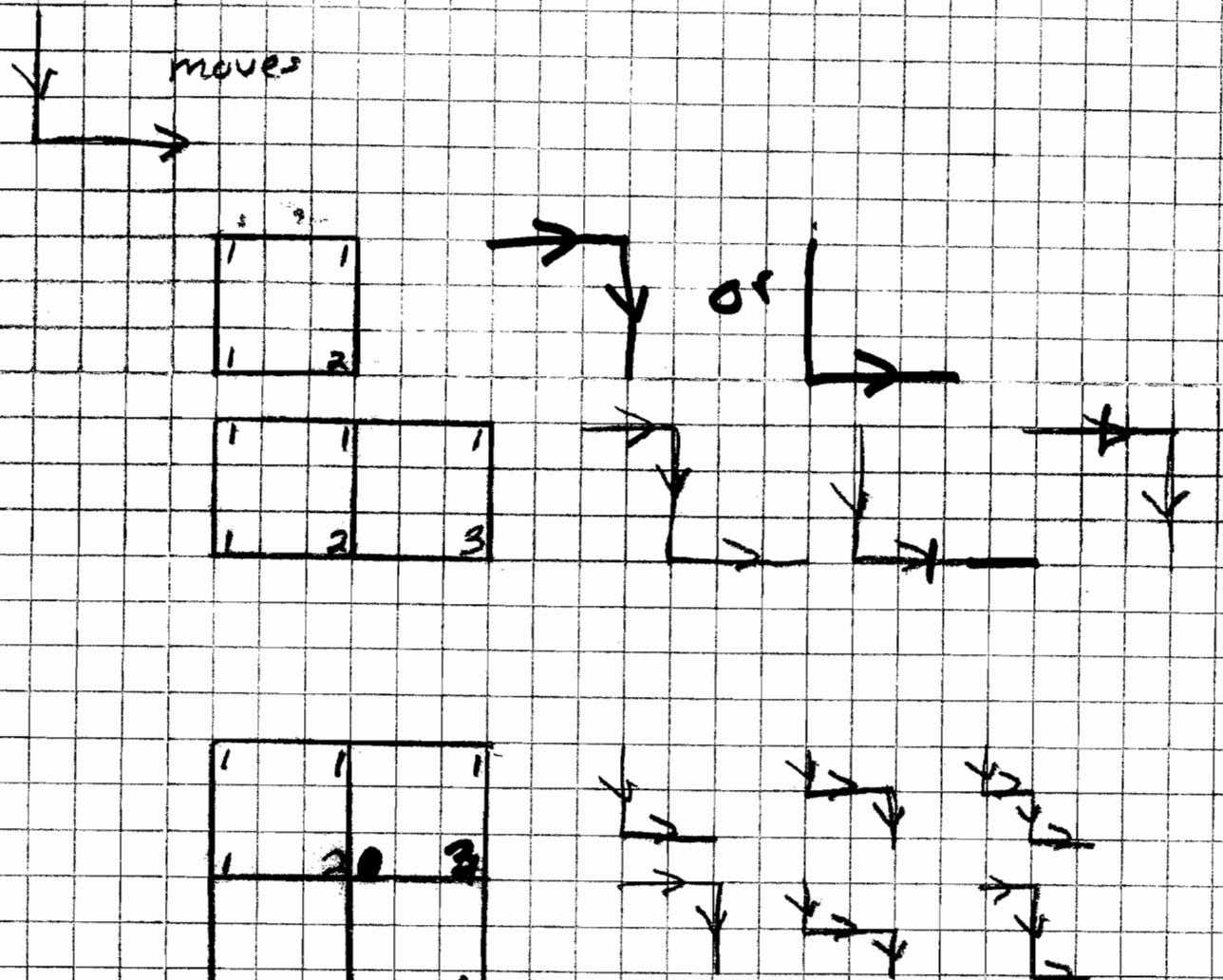
Probability of landing in the Irregular Shaped Side Regions.

$$1-\frac{25\pi}{100}=\frac{4}{4}-\frac{\pi}{4}=\frac{4-\pi}{4}\approx.21$$



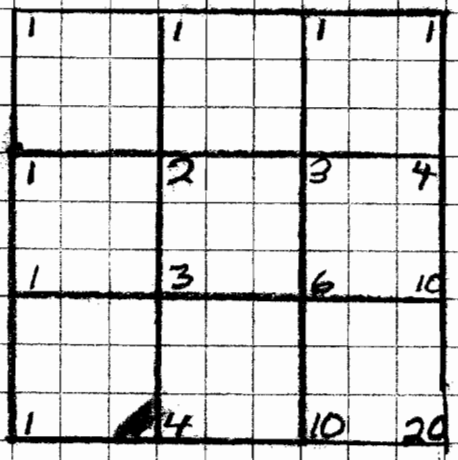

or .105 for each

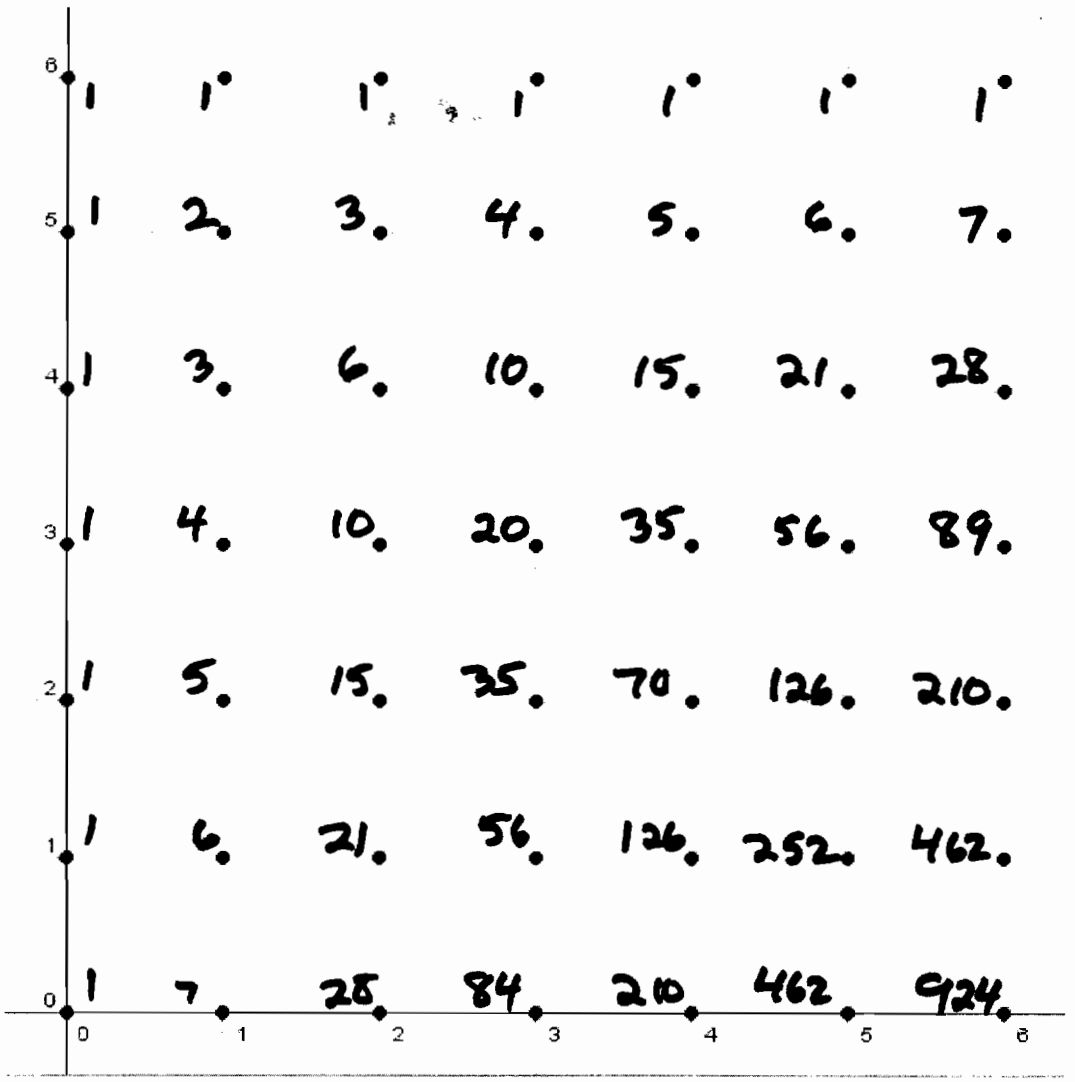
$A=\pi\cdot 5^2=25\pi$


moves
or


1 1 1
1 2 3
1 3 6




1 1 1 1
1 2 3 4
1 3 6 10
1 4 10 20

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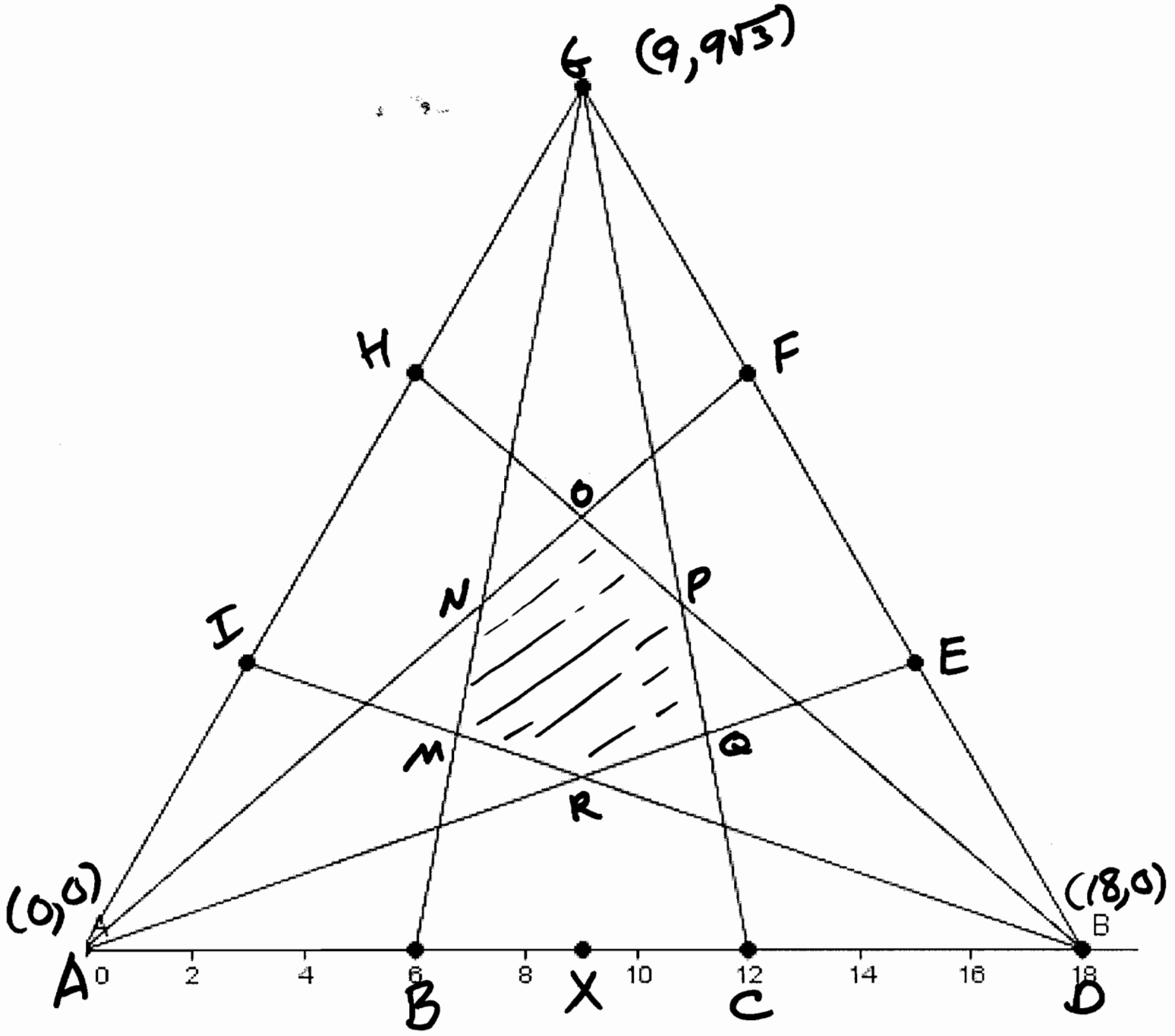


$AI = AB = BC = CD = DE = EF = FG = GH = HI =$

Find $\dfrac{\text{Area of hexagon } MNOPQR}{\text{Area of triangle } ADG}$

AG: $y=\sqrt{3}\,x$ DG: $y=-\sqrt{3}\,x+18\sqrt{3}$

DH: $y=\frac{-\sqrt{3}}{2}x+9\sqrt{3}$ DI: $y=\frac{-\sqrt{3}}{5}x+\frac{18\sqrt{3}}{5}$

AF: $y=\frac{\sqrt{3}}{5}x$ GB: $y=\sqrt{3}\,x-16\sqrt{3}$

GC: $y=-\sqrt{3}\,x+36\sqrt{3}$ GQ: $x=9$

$O(9,\frac{9\sqrt{3}}{2})$ $R(9,\frac{9\sqrt{5}}{2})$ $N(\frac{36}{5},\frac{18\sqrt{3}}{5})$

$M(\frac{27}{4},\frac{9\sqrt{3}}{4})$ $Q(\frac{45}{4},\frac{9\sqrt{3}}{4})$ $P(\frac{54}{5},\frac{18\sqrt{3}}{5})$

Area of Triangle AGD=$\frac{81}{2}\sqrt{3}$

Area of Triangle NOP=$\frac{81}{50}\sqrt{3}$--Section 1

Area of Triangle MRQ=$\frac{81}{80}\sqrt{3}$--Section 2

Area of Trapezoid MQPN=$\frac{2187}{400}\sqrt{3}$--Section 3

Section 1 + Section 2 + Section 3 = $\frac{3240}{400}\sqrt{3}=8.1\sqrt{3}$



So Ratio = $\frac{1}{10}$

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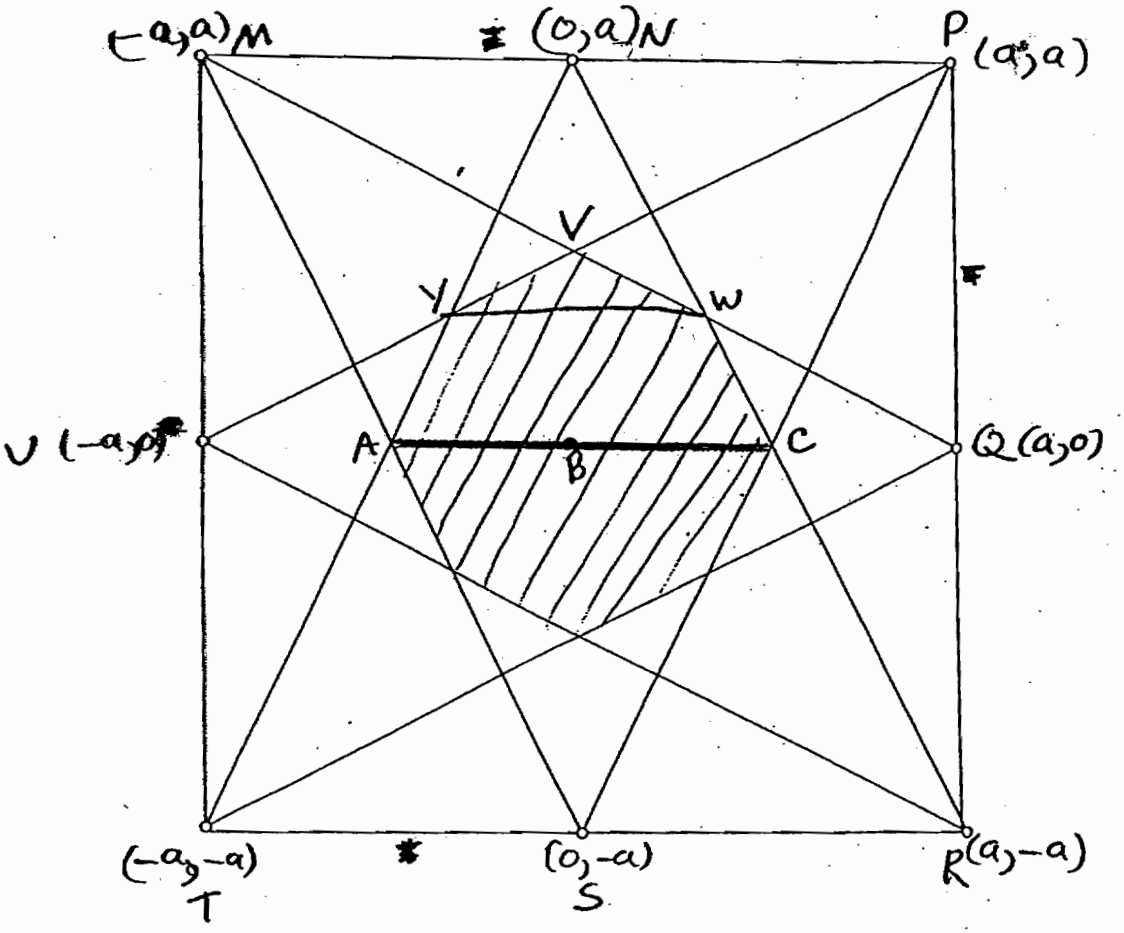
# AREA OF STRIPED REGION



MQ: $y= -\frac{1}{2}(x-a)$ PV: $y= \frac{1}{2}(x+a)$

NT: slope(m) = $\frac{a+a}{0+a}$ =2 so y-a = 2x

Intersection of MQ and PV (Point V)

$-\frac{1}{2}(x-a) = \frac{1}{2}(x+a)$

-x+a = x+ a so x=0 and $y=-\frac{1}{2}(-a)$ so $v(0, \frac{a}{2})$

Intersection of NR and MQ (Point W)

y-a = -2x

$y=-\frac{1}{2}(x-a)$

$a-2x = -\frac{1}{2}(x-a)$ so $x= \frac{a}{3}$

Then $y-a = -2(\frac{a}{3})$ and $y= a- \frac{2a}{3}$

$W(\frac{a}{3}, \frac{a}{3})$ and Point $C(\frac{a}{2}, 0)$

Area of Trapezoid ACWY = $\frac{1}{2}$(YW+AC)(WX)=

$\frac{1}{2}(\frac{2a}{3}+a)(\frac{a}{3})=\frac{1}{2}(\frac{5a}{3})(\frac{a}{3})=\frac{5a^2}{18}$

Area of Triangle YWX = $\frac{1}{2}(\frac{2a}{3})(\frac{a}{2}-\frac{a}{3})=$

$(\frac{a}{3})(\frac{a}{6})=(\frac{a^2}{18})$

Shaded Area = $2(\frac{5a^2}{18}+\frac{a^2}{18})=\frac{2a^2}{3}$

Area of Square = $(2a)^2 = 4a^2$

Ratio of Shaded Area of Total Area=

$(\frac{2a^2}{3})(\frac{1}{4a^2})=\frac{1}{6}$

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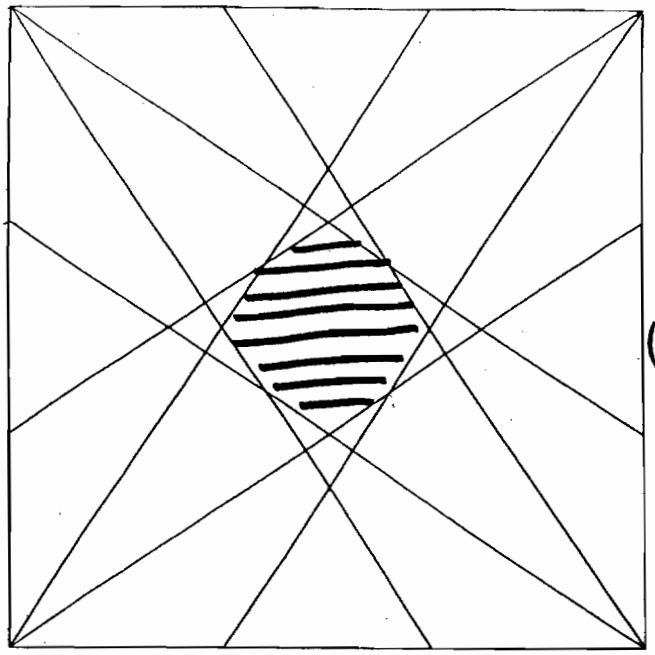
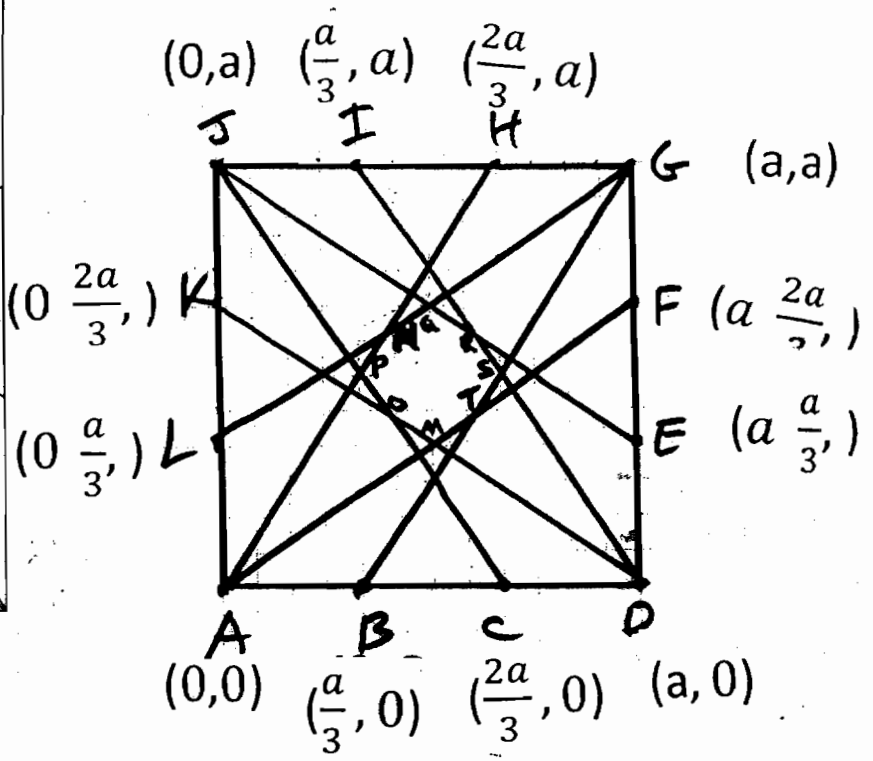




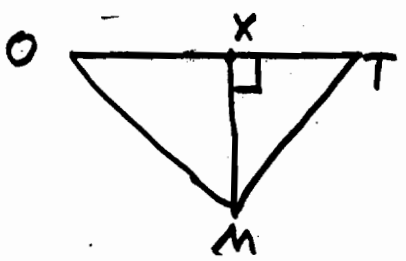

AF: $y=\frac{2}{3}x$ DK: $y=-\frac{2}{3}x+\frac{2a}{3}$ CJ: $y=-\frac{3}{2}x+a$

(AF)∩(DK) IS $M(\frac{a}{2}, \frac{a}{3})$

(DK)∩(CJ) IS $O(\frac{2a}{5}, \frac{2a}{5})$

$P(\frac{3a}{10}, \frac{a}{2})$ $N(\frac{2a}{5}, \frac{3a}{5})$ $Q(\frac{a}{2}, \frac{2a}{3})$ $R(\frac{2a}{5}, \frac{3a}{5})$ $S(\frac{7a}{10}, \frac{a}{2})$ $T(\frac{3a}{5}, \frac{2a}{5})$

$OT=\frac{3a}{5}-\frac{2a}{5}=\frac{a}{5}$ AND $QM=\frac{a}{3}$




$XM=(\frac{a}{3}-\frac{a}{5})\div 2=\frac{a}{15}$

AREA OF SQUARE ONRT $=\frac{a^2}{25}$

AREA OF 4 TRIANGLES $= 4\,(.5)(\frac{a}{5})(\frac{a}{15})=\frac{2a^2}{75}$

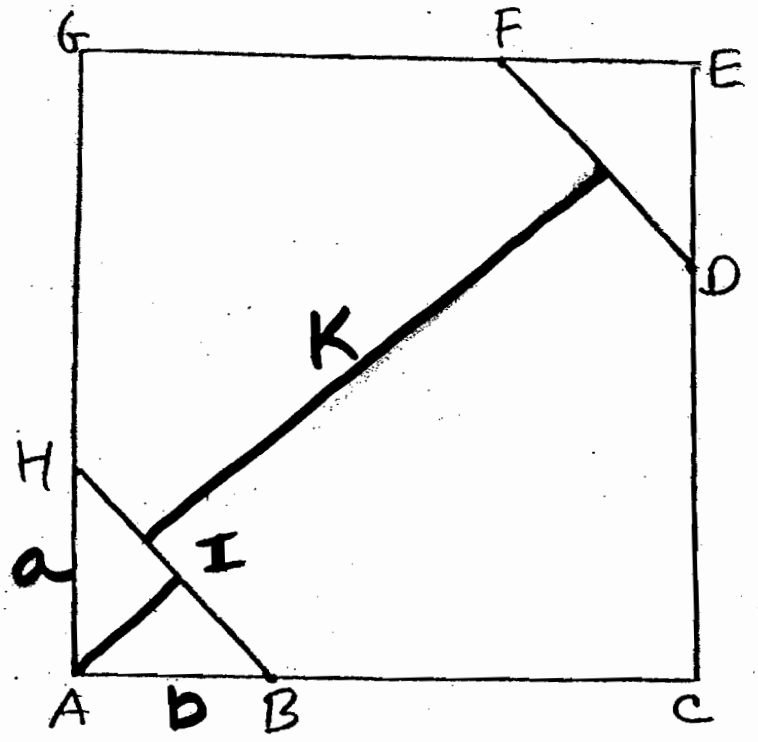
AREA OF (MNOPQRST) $=\frac{a^2}{25}+\frac{2a^2}{75}=\frac{a^2}{15}$

$$\frac{Octagonal\ Area}{Big\ Square\ Area}=\frac{\frac{a^2}{15}}{a^2}=\frac{1}{15}$$

## Lawn Cutting

ACEG is a Square. HB and DF are parallel.

Find the Probability of landing in Region GHBCDF if AG=100



U=percent cut  100-u=percent uncut

$\frac{ab}{2}=\frac{100-u}{2}$

Since a=b, $a^2$=100-u, then a=$\sqrt{100-u}$

$(HB)^2=(FD)^2$=(100-u)+(100-u)

HB=FD=$\sqrt{200-2u}$

$\frac{(HB)(AI)}{2}=\frac{100-u}{2}$

$$So\ AI=\frac{100-u}{\sqrt{200-u}}$$

So k= $\sqrt{200-2(\frac{100-u}{\sqrt{200-u}})}$

$$K=\sqrt{200}+\frac{2u-200}{\sqrt{200-2u}}$$

$$K=\sqrt{200}+\frac{2u-200}{\sqrt{200-2u}}$$

$$K=\sqrt{200}-\sqrt{200-2u}$$

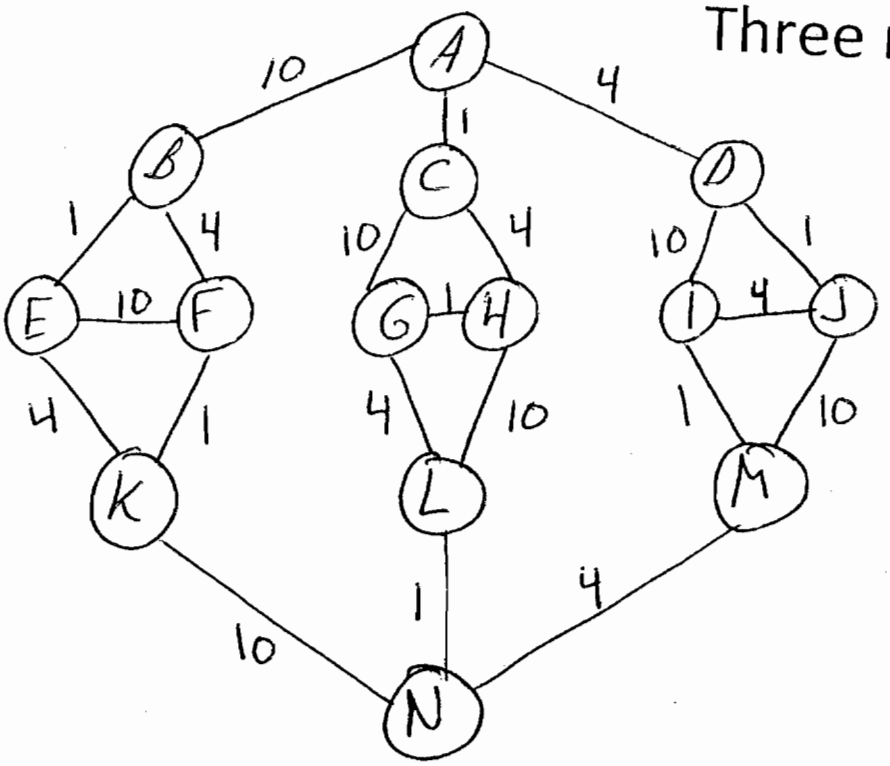
| U | k |
|---|---|
| 2 | .14 |
| 15.5 | 1.14 |
| 34.5 | 2.24 |
| 50 | 4.14 |
| 68 | 5.14 |
| 75.5 | 6.14 |
| 82 | 8.14 |
| 87.5 | 9.14 |
| 92 | 10.14 |
| 92.5 | 11.14 |
| 98 | 12.14 |
| 99.5 | 13.14 |
| 100 | 14.14=$\sqrt{200}$ |

a K

| x | y | triangle area | polygonal strip | percentage |
|---|---|---|---|---|
| 0 | 141.4213562 | 0 | 10000 | 1 |
| 5 | 134.3502884 | 25 | 9975 | 0.9975 |
| 10 | 127.2792206 | 100 | 9900 | 0.99 |
| 15 | 120.2081528 | 225 | 9775 | 0.9775 |
| 20 | 113.1370849 | 400 | 9600 | 0.96 |
| 25 | 106.0660172 | 625 | 9375 | 0.9375 |
| 30 | 98.99494937 | 900 | 9100 | 0.91 |
| 35 | 91.92388155 | 1225 | 8775 | 0.8775 |
| 40 | 84.85281374 | 1600 | 8400 | 0.84 |
| 45 | 77.78174593 | 2025 | 7975 | 0.7975 |
| 50 | 70.71067812 | 2500 | 7500 | 0.75 |
| 55 | 63.63961031 | 3025 | 6975 | 0.6975 |
| 60 | 56.56854249 | 3600 | 6400 | 0.64 |
| 65 | 49.49747468 | 4225 | 5775 | 0.5775 |
| 70 | 42.42640687 | 4900 | 5100 | 0.51 |
| 75 | 35.35533906 | 5625 | 4375 | 0.4375 |
| 80 | 28.28427125 | 6400 | 3600 | 0.36 |
| 85 | 21.21320344 | 7225 | 2775 | 0.2775 |
| 90 | 14.14213562 | 8100 | 1900 | 0.19 |
| 95 | 7.071067812 | 9025 | 975 | 0.0975 |
| 100 | 0 | 10000 | 0 | 0 |

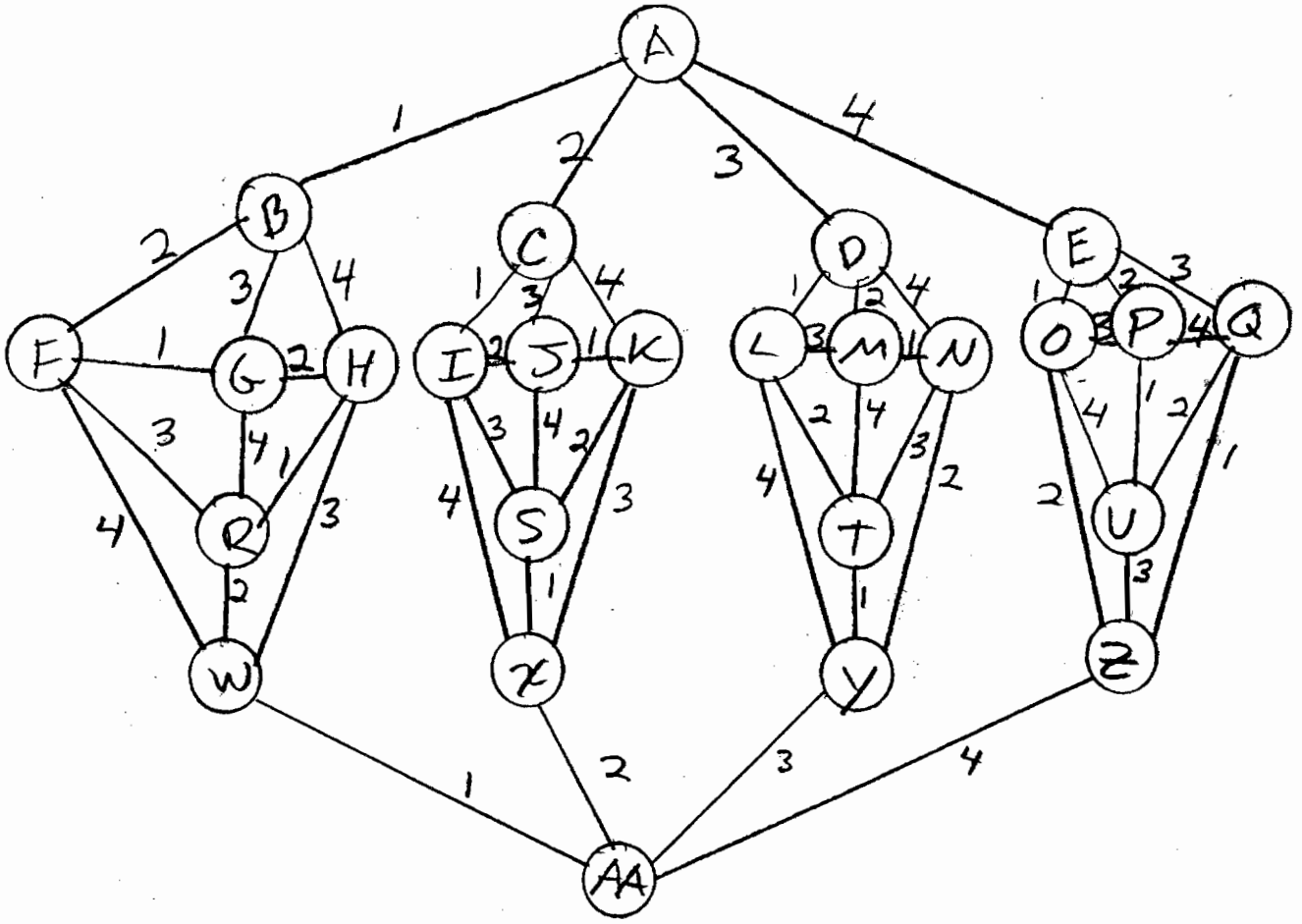
$a = 100\sqrt{1-p}$

# Three meeting chart

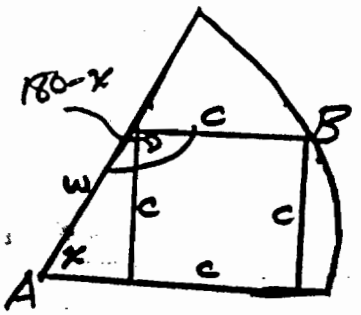



| 10-12 | 1-3 | 4-6 |
|---|---|---|
| A-B | A-C | A-D |
| E-F | G-H | I-J |
| K-N | L-N | N-M |
| C-G | B-E | B-F |
| H-L | F-K | E-K |
| D-I | I-M | G-L |
| J-M | D-J | C-H |

Four meeting chart




dsmith@dbq.edu
http://www.dbq.edu/library/facultypubs/DickSmith.cfm




$$\sin x = \frac{c}{w}$$

$$w = \frac{c}{\sin x}$$

$$AB^2 = 1 = w^2 + c^2 - 2wc\cos(180 - x)$$

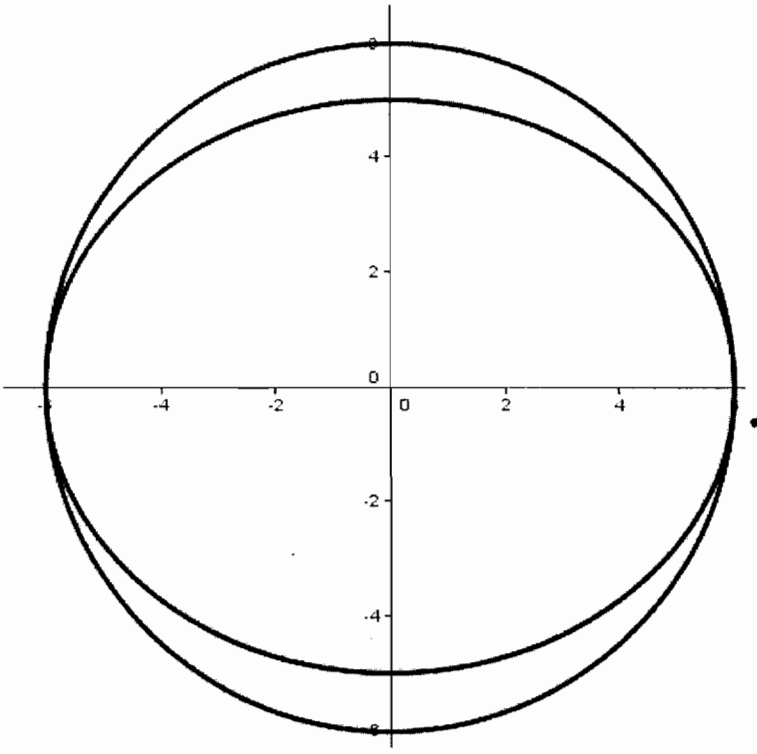
$$1 = \frac{c^2}{\sin^2 x} + c^2 - \frac{2c^2}{\sin x}(-\cos x)$$

$$1 = \frac{c^2(1 + \sin^2 x + 2\sin x \cos x)}{\sin^2 x}$$

Area of the square =

$$c^2 = \frac{\sin^2 x}{\sin^2 x + \sin 2x + 1}$$

| Angle x | sin-squared | sin2x | c-squared | sector A | square/sector |
|---|---|---|---|---|---|
| 10 | 0.03015364 | 0.34202 | 0.021975 | 0.087266 | 0.251816219 |
| 20 | 0.11697759 | 0.642787 | 0.066473 | 0.174533 | 0.380864989 |
| 30 | 0.24999962 | 0.866025 | 0.118146 | 0.261799 | 0.451284457 |
| 40 | 0.41317533 | 0.984808 | 0.172301 | 0.349066 | 0.493606999 |
| 50 | 0.58682336 | 0.984808 | 0.228191 | 0.436332 | 0.522975894 |
| 60 | 0.74999923 | 0.866026 | 0.286694 | 0.523598 | 0.547545971 |
| 70 | 0.88302156 | 0.642789 | 0.349599 | 0.610865 | 0.572302251 |
| 80 | 0.96984591 | 0.342022 | 0.419507 | 0.698131 | 0.600900633 |
| 90 | 1 | 2.65E-06 | 0.499999 | 0.785398 | 0.63661 |
| | | | Square | Sector | |




Circle

$x^2+y^2=36$

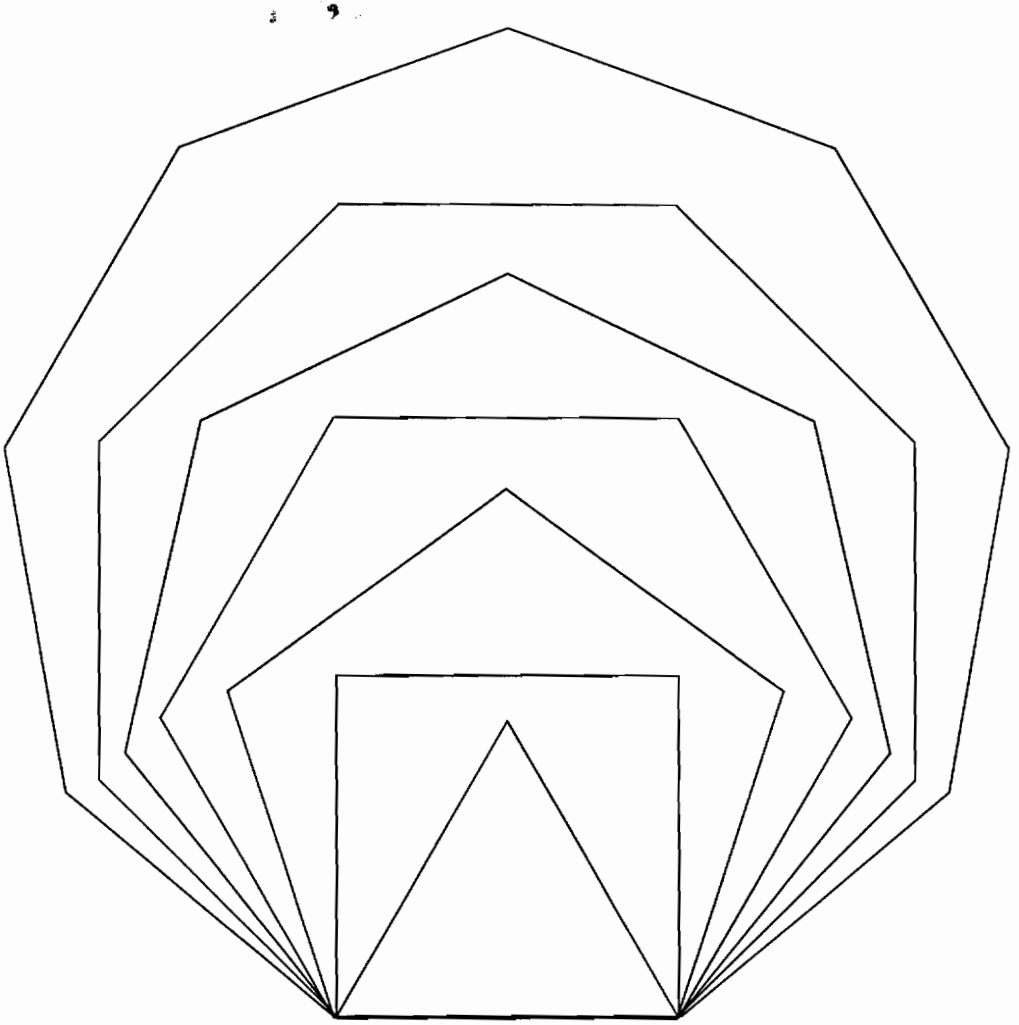
Ellipse

$\frac{x^2}{36}+\frac{y^2}{25}=1$

Area of Circle = $36\pi = \pi r^2$

Area of Ellipse = $30\pi = \pi ab$

Probability of landing in the Ellipse = $\frac{5}{6}$



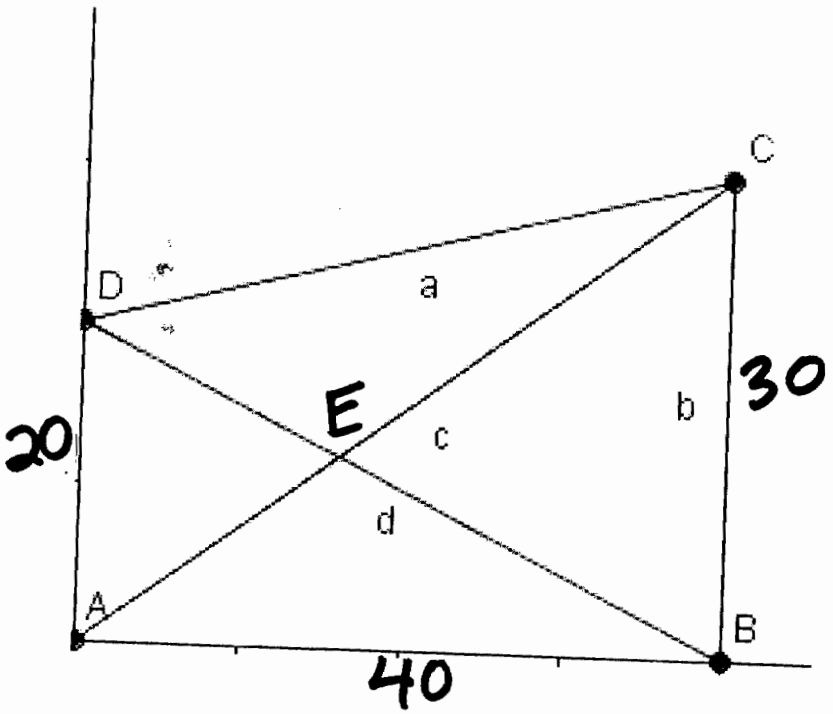
| side | n | p | ANGLE | TANGENT | APOTHEM | AREA OF POLY | AREA OF CIRCLE | PROBABILITY |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 3 | 60 | 0.57735 | 0.28867 | 0.43301 | 0.26180 | 0.60460 |
| 1 | 4 | 4 | 90 | 1 | 0.5 | 1 | 0.78540 | 0.78540 |
| 1 | 5 | 5 | 108 | 1.37638 | 0.68819 | 1.72047 | 1.48787 | 0.86480 |
| 1 | 6 | 6 | 120 | 1.73205 | 0.86602 | 2.59807 | 2.35618 | 0.90690 |
| 1 | 7 | 7 | 128.57143 | 2.07652 | 1.03826 | 3.63390 | 3.38657 | 0.93194 |
| 1 | 8 | 8 | 135 | 2.41421 | 1.20710 | 4.82841 | 4.57761 | 0.94806 |
| 1 | 9 | 9 | 140 | 2.74747 | 1.37373 | 6.18180 | 5.92864 | 0.95905 |
| 1 | 10 | 10 | 144 | 3.07767 | 1.53884 | 7.69418 | 7.43934 | 0.96688 |
| 1 | 11 | 11 | 147.27273 | 3.40567 | 1.70284 | 9.36560 | 9.10952 | 0.97266 |
| 1 | 12 | 12 | 150 | 3.73203 | 1.86602 | 11.19610 | 10.93908 | 0.97704 |
| 1 | 13 | 13 | 152.30769 | 4.05714 | 2.02857 | 13.18570 | 12.92794 | 0.98045 |
| 1 | 14 | 14 | 154.28571 | 4.38126 | 2.19063 | 15.33442 | 15.07607 | 0.98315 |
| 1 | 15 | 15 | 156 | 4.70460 | 2.35230 | 17.64226 | 17.38343 | 0.98533 |
| 1 | 16 | 16 | 157.5 | 5.02731 | 2.51365 | 20.10924 | 19.85001 | 0.98711 |
| 1 | 17 | 17 | 158.82353 | 5.34949 | 2.67475 | 22.73534 | 22.47578 | 0.98858 |
| 1 | 18 | 18 | 160 | 5.67124 | 2.83562 | 25.52059 | 25.26073 | 0.98982 |
| 1 | 19 | 19 | 161.05263 | 5.99263 | 2.99631 | 28.46498 | 28.20487 | 0.99086 |
| 1 | 20 | 20 | 162 | 6.31370 | 3.15685 | 31.56851 | 31.30818 | 0.99175 |
| 1 | 21 | 21 | 162.85714 | 6.63451 | 3.31726 | 34.83119 | 34.57065 | 0.99252 |
| 1 | 22 | 22 | 163.63636 | 6.95509 | 3.47755 | 38.25301 | 37.99229 | 0.99318 |
| 1 | 23 | 23 | 164.34783 | 7.27547 | 3.63774 | 41.83398 | 41.57308 | 0.99376 |
| 1 | 24 | 24 | 165 | 7.59568 | 3.79784 | 45.57410 | 45.31303 | 0.99427 |
| 1 | 25 | 25 | 165.6 | 7.91574 | 3.95787 | 49.47336 | 49.21214 | 0.99472 |
| 1 | 26 | 26 | 166.15385 | 8.23566 | 4.11783 | 53.53177 | 53.27040 | 0.99512 |
| 1 | 27 | 27 | 166.66667 | 8.55546 | 4.27773 | 57.74933 | 57.48781 | 0.99547 |
| 1 | 28 | 28 | 167.14286 | 8.87515 | 4.43757 | 62.12603 | 61.86438 | 0.99579 |
| 1 | 29 | 29 | 167.58621 | 9.19474 | 4.59737 | 66.66188 | 66.40009 | 0.99607 |
| 1 | 30 | 30 | 168 | 9.51425 | 4.75713 | 71.35688 | 71.09495 | 0.99633 |
| 1 | 31 | 31 | 168.38710 | 9.83368 | 4.91684 | 76.21103 | 75.94895 | 0.99656 |
| 1 | 32 | 32 | 168.75 | 10.15304 | 5.07652 | 81.22433 | 80.96210 | 0.99677 |
| 1 | 33 | 33 | 169.09091 | 10.47234 | 5.23617 | 86.39677 | 86.13440 | 0.99696 |
| 1 | 34 | 34 | 169.41176 | 10.79157 | 5.39579 | 91.72836 | 91.46584 | 0.99714 |
| 1 | 35 | 35 | 169.71429 | 11.11075 | 5.55538 | 97.21910 | 96.95643 | 0.99730 |
| 1 | 36 | 36 | 170 | 11.42989 | 5.71494 | 102.86899 | 102.60616 | 0.99744 |
| 1 | 37 | 37 | 170.27027 | 11.74898 | 5.87449 | 108.67802 | 108.41503 | 0.99758 |
| 1 | 38 | 38 | 170.52632 | 12.06802 | 6.03401 | 114.64620 | 114.38304 | 0.99770 |
| 1 | 39 | 39 | 170.76923 | 12.38703 | 6.19351 | 120.77353 | 120.51019 | 0.99782 |
| 1 | 40 | 40 | 171 | 12.706 | 6.353 | 127.06 | 126.79649 | 0.99793 |
| 1 | 41 | 41 | 171.21951 | 13.02494 | 6.51247 | 133.50562 | 133.24192 | 0.99802 |
| 1 | 42 | 42 | 171.42857 | 13.34385 | 6.67192 | 140.11039 | 139.84649 | 0.99812 |
| 1 | 43 | 43 | 171.62791 | 13.66273 | 6.83136 | 146.8743 | 146.61021 | 0.99820 |
| 1 | 44 | 44 | 171.81818 | 13.98158 | 6.99079 | 153.79736 | 153.53306 | 0.99828 |
| 1 | 45 | 45 | 172 | 14.30041 | 7.15020 | 160.87956 | 160.61505 | 0.99836 |
| 1 | 46 | 46 | 172.17391 | 14.61921 | 7.30961 | 168.12092 | 167.85618 | 0.99843 |
| 1 | 47 | 47 | 172.34043 | 14.93799 | 7.46900 | 175.52141 | 175.25644 | 0.99849 |

How many Triangles?

$_5C_3 - 3(_3C_2)$

$10 - 2 = 8$

Probabilities?




Area(ABCD)=.5(40)(20+30)=1000

1) Area(ABC)=.5(30)(40)=600

2) Area(DAB)=.5(20)(40)=400

(Line c) $y=-\frac{1}{2}x+20$ (Line d) $y=\frac{3}{4}x$

$\frac{3}{4}x=-\frac{1}{2}x+20$ so x=16 and y=12

3) Area (AEB)= .5(40)(12)=240

4) Area (BCE) =.5(30)(24)=360

Check 240+360=600

5) Area(ADE)=.5(20)(16)=160

Check 240+160=400

6) So Area(DEC)=1000-240-360-160=240

So Probability of being in the right triangle is .36,

Probability of being in the left triangle is .16,

Probability of being in the upper triangle is .24, and

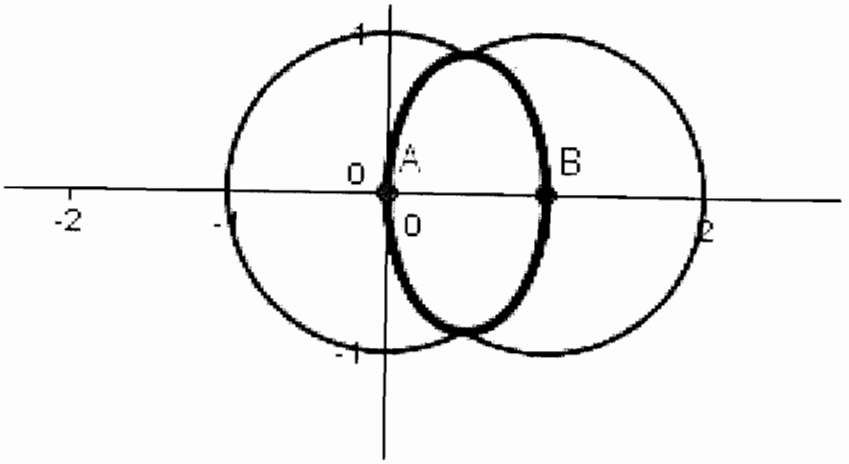
Probability of being in the lower triangle is .24.

4 partitions

7) Area(ACD) = Area(BAD) = 400

8) Area(BCD) = Area(BCE) = 600

# Probability of being in the elliptical intersection of two unit circles




$x^2+y^2=1$ is left circle

$x^2+(y-1)^2=1$ is the right circle

Let the distance between the centers =AB=a

To find the intersection of the two circles let x=$\frac{a}{2}$

So y=$\sqrt{1-(\frac{a}{2})^2}$

If AB=a=1, y=$\sqrt{1-(\frac{1}{2})^2}$ =$\sqrt{.75}$ =.866

The semi-minor axis =.5 and the semi-major axis =.866

The area of the ellipse is $\pi(.5)(.866) = 1.36$

The area of the whole region =2($\pi$)$1^2$-1.36=4.922

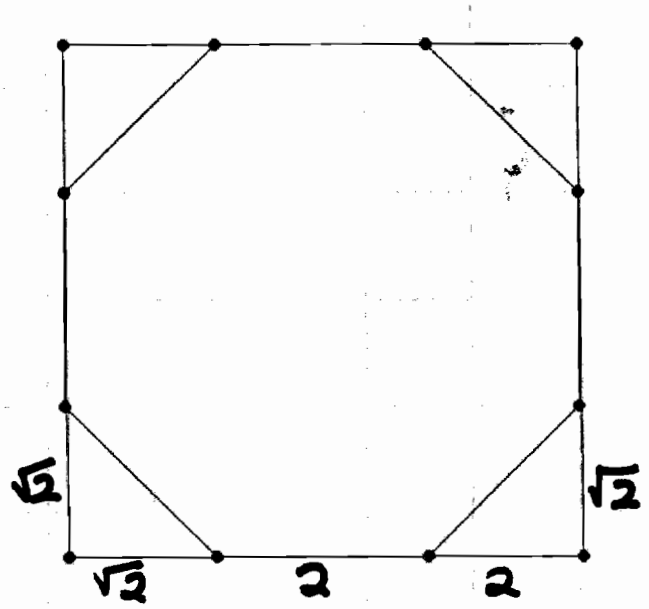
The probability of landing in the ellipse =$\frac{1.36}{4.922}$ = .2763

| distance between centers | semi-minor axis | semi-major axis | area of ellipse | area of whole region | probability of being in ellipse |
|---|---|---|---|---|---|
| 2 | 0 | 0.707106781 | 0 | 6.28318 | 0 |
| 1.95 | 0.025 | 0.715891053 | 0.056225904 | 6.226954096 | 0.009029439 |
| 1.9 | 0.05 | 0.724568837 | 0.113814911 | 6.169365089 | 0.018448399 |
| 1.85 | 0.075 | 0.733143915 | 0.172742819 | 6.110437181 | 0.028270124 |
| 1.8 | 0.1 | 0.741619849 | 0.23298655 | 6.05019345 | 0.038508942 |
| 1.75 | 0.125 | 0.75 | 0.294524063 | 5.988655938 | 0.049180328 |
| 1.7 | 0.15 | 0.758287544 | 0.357334285 | 5.925845715 | 0.060300977 |
| 1.65 | 0.175 | 0.766485486 | 0.421397049 | 5.861782951 | 0.071888886 |
| 1.6 | 0.2 | 0.774596669 | 0.48669303 | 5.79648697 | 0.083963448 |
| 1.55 | 0.225 | 0.782623792 | 0.553203693 | 5.729976307 | 0.096545546 |
| 1.5 | 0.25 | 0.790569415 | 0.620911242 | 5.662268758 | 0.109657678 |
| 1.45 | 0.275 | 0.798435971 | 0.689798577 | 5.593381423 | 0.123324073 |
| 1.4 | 0.3 | 0.806225775 | 0.75984925 | 5.52333075 | 0.13757084 |
| 1.35 | 0.325 | 0.81394103 | 0.831047425 | 5.452132575 | 0.152426122 |
| 1.3 | 0.35 | 0.821583836 | 0.903377847 | 5.379802153 | 0.167920273 |
| 1.25 | 0.375 | 0.829156198 | 0.976825807 | 5.306354193 | 0.184086054 |
| 1.2 | 0.4 | 0.836660027 | 1.051377109 | 5.231802891 | 0.200958853 |
| 1.15 | 0.425 | 0.844097151 | 1.127018046 | 5.156161954 | 0.218576929 |
| 1.1 | 0.45 | 0.851469318 | 1.203735373 | 5.079444627 | 0.23698169 |
| 1.05 | 0.475 | 0.858778202 | 1.28151628 | 5.00166372 | 0.256218001 |
| 1 | 0.5 | 0.866025404 | 1.360348374 | 4.922831626 | 0.276334532 |
| 0.95 | 0.525 | 0.87321246 | 1.440219654 | 4.842960346 | 0.297384152 |
| 0.9 | 0.55 | 0.880340843 | 1.521118494 | 4.762061506 | 0.31942437 |
| 0.85 | 0.575 | 0.887411967 | 1.603033624 | 4.680146376 | 0.342517839 |
| 0.8 | 0.6 | 0.894427191 | 1.685954111 | 4.597225889 | 0.366732928 |
| 0.75 | 0.625 | 0.901387819 | 1.769869349 | 4.513310651 | 0.392144367 |
| 0.7 | 0.65 | 0.908295106 | 1.854769035 | 4.428410965 | 0.41883399 |
| 0.65 | 0.675 | 0.915150261 | 1.940643163 | 4.342536837 | 0.446891583 |
| 0.6 | 0.7 | 0.921954446 | 2.027482007 | 4.255697993 | 0.476415857 |
| 0.55 | 0.725 | 0.928708781 | 2.115276109 | 4.167903891 | 0.507515568 |
| 0.5 | 0.75 | 0.935414347 | 2.204016268 | 4.079163732 | 0.540310812 |
| 0.45 | 0.775 | 0.942072184 | 2.293693528 | 3.989486472 | 0.57493453 |
| 0.4 | 0.8 | 0.948683298 | 2.38429917 | 3.89888083 | 0.611534251 |
| 0.35 | 0.825 | 0.955248659 | 2.475824698 | 3.807355302 | 0.650274141 |
| 0.3 | 0.85 | 0.961769203 | 2.568261834 | 3.714918166 | 0.691337391 |
| 0.25 | 0.875 | 0.968245837 | 2.661602508 | 3.621577492 | 0.734929051 |
| 0.2 | 0.9 | 0.974679434 | 2.755838848 | 3.527341152 | 0.781279363 |
| 0.15 | 0.925 | 0.981070844 | 2.850963175 | 3.432216825 | 0.830647748 |
| 0.1 | 0.95 | 0.987420883 | 2.946967993 | 3.336212007 | 0.883327554 |
| 0.05 | 0.975 | 0.993730346 | 3.043845984 | 3.239334016 | 0.939651783 |
| 0 | 1 | 1 | 3.14159 | 3.14159 | 1 |

Ratio of the area of a stop sign to the area of its square "stock"

Area of the four corner triangles

$$4\left(\frac{\sqrt{2}^2}{2}\right)=4$$




Area of "stock square"

$$(2+2\sqrt{2})^2 = 4+8\sqrt{2}+8=12+8\sqrt{2}$$

Ratio

$$\frac{(12+8\sqrt{2})-4}{12+8\sqrt{2}}$$

$$\frac{8+8\sqrt{2}}{12+8\sqrt{2}}$$

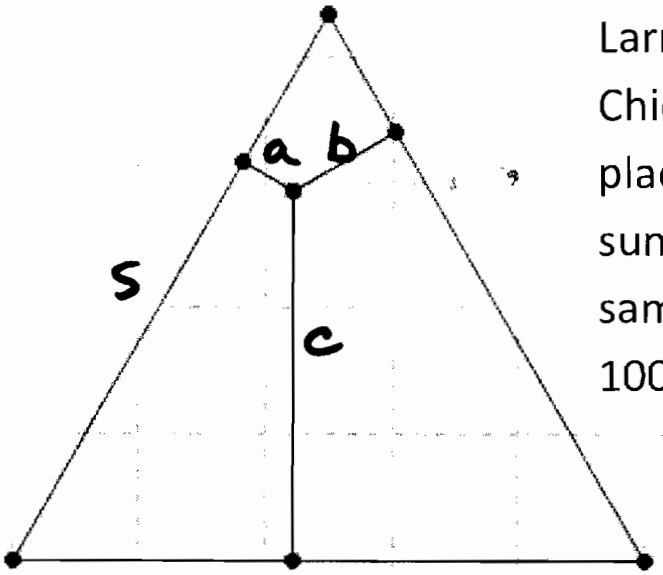
$$\frac{2+2\sqrt{2}}{3+2\sqrt{2}}$$

$$\left(\frac{2+2\sqrt{2}}{3+2\sqrt{2}}\right)\left(\frac{3-2\sqrt{2}}{3-2\sqrt{2}}\right)$$

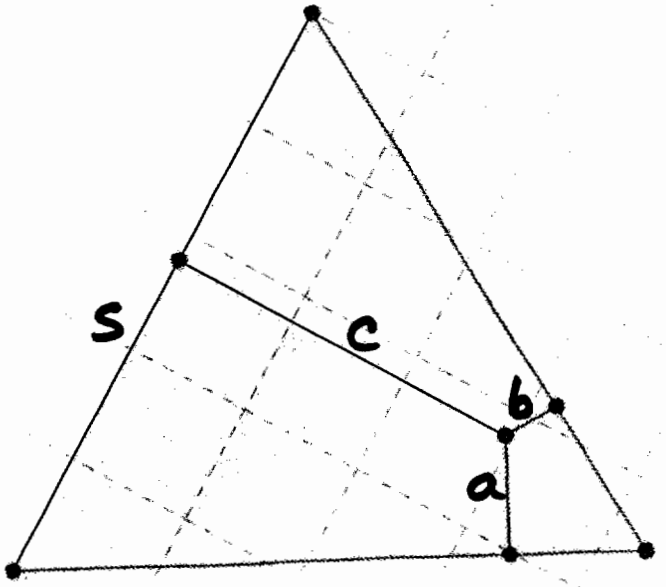
$$\frac{6-4\sqrt{2}+6\sqrt{2}-8}{9-8}$$

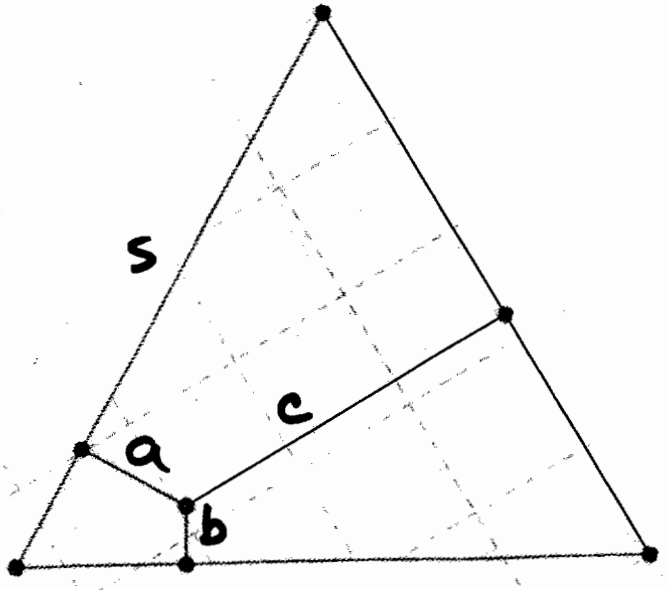
$$2\sqrt{2}-2$$

$$.8284$$




Larry Copes and Jeremy Kahan from Chicago proved that no matter where you place a point in an equilateral triangle, the sum of the distances to the sides is the same. See Mathematics Teacher Volume 100 August 2006.

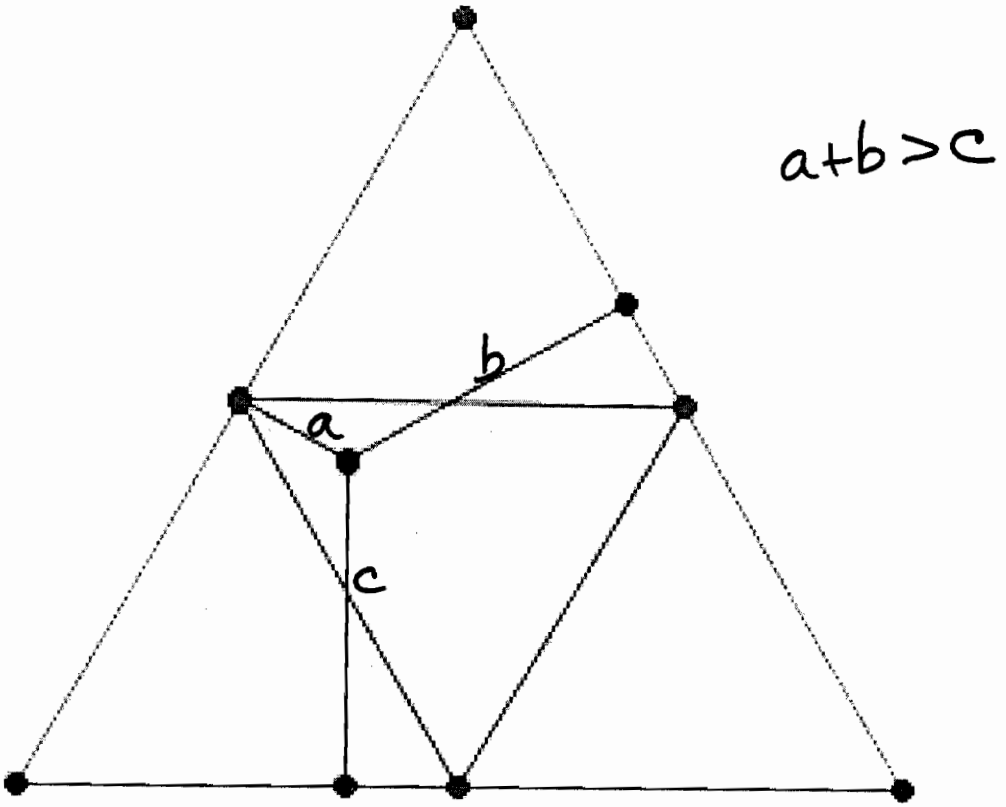



in each triangle

$$a+b+c=\frac{s\sqrt{3}}{2}$$

In the Mathematics Teacher Volume 101 March 2008, I tied their idea into cutting a string into three pieces of arbitrary length a, b and c. The drawings demonstrate that the probability of choosing three random cuts to form a triangle is 1/4



$$a^2 + b^2 = c^2 + d^2$$

$$a^2 - c^2 = d^2 - b^2$$

(a+c)(a-c)=(d+b)(d-b)

a>b and c>d

If a=10 and c=6,

(16)(4)=(32)(2) =(d+b)(d-b)

Then b=15 and d=17

$10^2 + 15^2 = 6^2 + 17$=325

$$a^2 + b^2 = c^2 + d^2$$

$$a^2 - c^2 = d^2 - b^2$$

(a+c)(a-c)=(d+b)(d-b)

a>b and c>d

If a=12 and c=8,

(20)(4)=(10)(8) =(d+b)(d-b)

Then b=1 and d=9

$12^2 + 1^2 = 8^2 + 9^2 = 145$

$$a^2 + b^2 + c^2 = d^2 + e^2 + f^2$$

$$a^2 - d^2 + b^2 - e^2 = f^2 - c^2$$

(a+d)(a-d)+(b+e)(b-e)=(f+c)(f-c)

a>d and b>e and f>c

If a=12, b=4, d=9 and e=2,

(21)(3)+(6)(2)=75=(25)(3) =(f+c)(f-c)

Then c=11 and f=14

$12^2 + 4^2 + 11^2 = 9^2 + 2^2 + 14^2 = 281$

Also (21)(3)+(6)(2)=75=(15)(5) =(f+c)(f-c)

Then c=5 and f=10

$12^2 + 4^2 + 5^2 = 9^2 + 2^2 + 10^2 = 185$

# Equal Arcs, Triangles, and Probability

MATHEMATICS TEACHER
Vol 96 No 9
December 2003

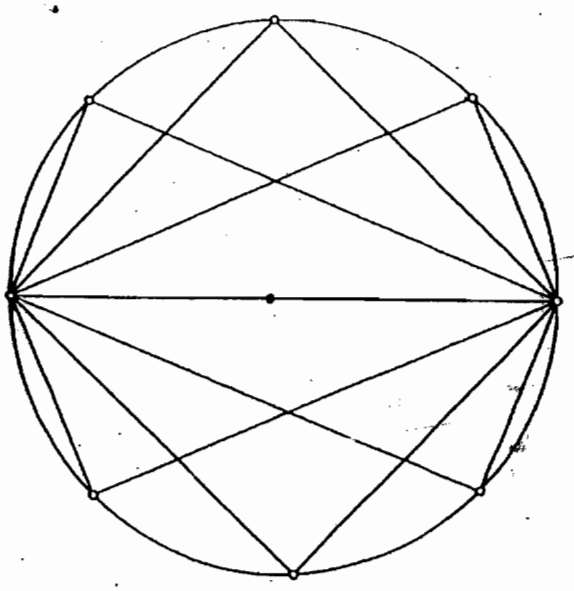


Fig. 4
Right triangles formed using only one diameter when $n$ is 8

$n$ = number of equally spaced points on the circumference of a circle
$r$ = number of right triangles
$t$ = total number of triangles
$p$ = ratio of $r$ to $t$

| $n$ | $r$ | $t$ | $\frac{r}{t}$ or $p$ |
|---|---|---|---|
| 4 | 4 | 4 | $\frac{4}{4}$ or $\frac{3}{3}$ |
| 6 | 12 | 20 | $\frac{12}{20}$ or $\frac{3}{5}$ |
| 8 | 24 | 56 | $\frac{24}{56}$ or $\frac{3}{7}$ |
| 10 | 40 | 120 | $\frac{40}{120}$ or $\frac{3}{9}$ |
| 12 | 60 | 220 | $\frac{60}{220}$ or $\frac{3}{11}$ |

Proof

$$\frac{\frac{n(n-2)}{2}}{C(n,3)} = \frac{\frac{n(n-2)}{2}}{\frac{n!}{(n-3)!3!}}$$

$$= \frac{n(n-2)(n-3)!(3)(2)(1)}{2n(n-1)(n-2)(n-3)!}$$

$$= \frac{3}{n-1}$$

1)Consider a sphere with radius (R) centered at the origin.

2)Drill a core with radius (c)

3)$x^2 + y^2 = R^2$ so x= $\sqrt{R^2 - y^2}$

4)Each Washer has an area of $\pi((\sqrt{R^2 - y^2})^2 - Rc^2$

5)V= $\pi$ ($R^2 - y^2 - c^2$)y

6) $-\sqrt{R^2 - c^2} < y < \sqrt{R^2 - c^2}$

7) *Volume of what is left* after drilling

$$= \pi \int_{-\sqrt{R^2-c^2}=-w}^{\sqrt{R^2-c^2}=w} (R^2 - y^2 - c^2)\, dy$$

$$=\pi \left(R^2 y - \frac{y^3}{3} - c^2 y\right)\Big|_{-w}^{w}$$

$$= \pi\left(\left(R^2 w - \frac{w^3}{3} - c^2 w\right) - \left(-R^2 w + \frac{w^3}{3} + c^2 w\right)\right)$$

$$= \pi\left(2R^2 w - \frac{2w^3}{3} - 2c^2 w\right)$$

$$=2\pi\, w\left(R^2 - \frac{w^2}{3} - c^2\right)$$

$$=2\,\pi\,\sqrt{R^2-\,c^2}\,(R^2-\frac{R^2}{3}+\frac{c^2}{3}-c^2)$$

$$=2\,\pi\,\sqrt{R^2-\,c^2}(\frac{2R^2-2c^2}{3})$$

$$=(\frac{4\pi}{3})(R^2-\,c^2)^{\frac{3}{2}}$$

So the Volume of the Core

$$=(\frac{4\pi}{3})(R^3-(R^2-\,c^2)^{\frac{3}{2}})$$

| R | c | Core V | Sphere V | Probability in Core |
|---|---|---|---|---|
| 10 | 0 | ~~2.85726E-12~~ | 4188.786667 | ~~6.82121E-13~~ 0 |
| 10 | 0.25 | 3.926373844 | 4184.860293 | 0.000937354 |
| 10 | 0.5 | 15.69812844 | 4173.088538 | 0.003747655 |
| 10 | 0.75 | 35.29313987 | 4153.493527 | 0.008425624 |
| 10 | 1 | 62.67445771 | 4126.112209 | 0.014962437 |
| 10 | 1.25 | 97.79018804 | 4090.996479 | 0.023345707 |
| 10 | 1.5 | 140.5733275 | 4048.213339 | 0.033559438 |
| 10 | 1.75 | 190.9415463 | 3997.84512 | 0.045583975 |
| 10 | 2 | 248.7969164 | 3939.98975 | 0.059395939 |
| 10 | 2.25 | 314.0255826 | 3874.761084 | 0.074968149 |
| 10 | 2.5 | 386.4973696 | 3802.289297 | 0.092269528 |
| 10 | 2.75 | 466.065321 | 3722.721346 | 0.111264993 |
| 10 | 3 | 552.5651617 | 3636.221505 | 0.131915327 |
| 10 | 3.25 | 645.8146749 | 3542.971992 | 0.154177027 |
| 10 | 3.5 | 745.6129842 | 3443.173682 | 0.178002138 |
| 10 | 3.75 | 851.7397258 | 3337.046941 | 0.203338053 |
| 10 | 4 | 963.9540957 | 3224.832571 | 0.230127283 |
| 10 | 4.25 | 1081.993752 | 3106.792915 | 0.258307199 |
| 10 | 4.5 | 1205.573549 | 2983.213118 | 0.287809727 |
| 10 | 4.75 | 1334.384068 | 2854.402598 | 0.318561 |
| 10 | 5 | 1468.089918 | 2720.696748 | 0.350480947 |
| 10 | 5.25 | 1606.327743 | 2582.458924 | 0.383482825 |
| 10 | 5.5 | 1748.703885 | 2440.082782 | 0.417472654 |
| 10 | 5.75 | 1894.791626 | 2293.995041 | 0.452348562 |
| 10 | 6 | 2044.127893 | 2144.658773 | 0.488 |
| 10 | 6.25 | 2196.209304 | 1992.577363 | 0.524306793 |
| 10 | 6.5 | 2350.487352 | 1838.299315 | 0.561137995 |
| 10 | 6.75 | 2506.362488 | 1682.424178 | 0.598350474 |
| 10 | 7 | 2663.176737 | 1525.609929 | 0.63578715 |
| 10 | 7.25 | 2820.204324 | 1368.582342 | 0.673274757 |
| 10 | 7.5 | 2976.639562 | 1212.147105 | 0.71062095 |
| 10 | 7.75 | 3131.580821 | 1057.205845 | 0.747610483 |
| 10 | 8 | 3284.008747 | 904.77792 | 0.784 |
| 10 | 8.25 | 3432.75564 | 756.0310264 | 0.819510735 |
| 10 | 8.5 | 3576.460634 | 612.3260328 | 0.853817804 |
| 10 | 8.75 | 3713.500505 | 475.2861621 | 0.886533691 |
| 10 | 9 | 3841.875209 | 346.9114577 | 0.91718092 |
| 10 | 9.25 | 3958.999252 | 229.7874143 | 0.945142249 |
| 10 | 9.5 | 4061.261715 | 127.5249512 | 0.969555635 |
| 10 | 9.75 | 4142.829959 | 45.95670736 | 0.989028635 |
| 10 | 10 | 4188.786667 | 0 | 1 |